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Issue Theme:
Load and Resistance Factor Design – Update for Wood

Editorial

The early 1990s saw a scramble to develop background procedures that would support the incorporation of load and resistance design procedures into design specifications for wood construction. Just a few years earlier, in 1980, the National Bureau of Standards published Special Publication 577 – which provided the basis for the load factors and load combinations that first appeared in ANSI A58.1-1982 (and subsequently ASCE 7). The AISC Steel Specification published its first LRFD Manual in 1986. Procedures for wood construction built upon the ASCE Pre-Standard Report (1988), ASTM D5457 (1993), and AF&PA-ASCE-16 (1995) – and were published in 1996 in the AF&PA LRFD Design Manual Package.

Subsequent editions of the *National Design Specification® (NDS®) for Wood Construction* harmonized many of the original LRFD provisions into today's NDS with very few modifications.

However, during the recent revisions of ASCE 7, several changes have been introduced that require reevaluation of D5457's reliability-based procedures. The most obvious change is the broadening acceptance of simplified closed-form equations for LRFD benchmarking (supplementing or replacing the more complex software-based methods). A second change, more likely to affect LRFD over the long-term, is the concept of designing to meet specific targets for reliability indices (rather than interpreting calculated reliability indices as broad indicators of relative performance across design scenarios). While viewed by some as an advancement, targeting a specific reliability index has its drawbacks when results vary significantly depending on analysis assumptions or differ from historically successful designs.

The four papers in this issue of Wood Design Focus examine the past, present, and potential future of reliability-based wood design. The first paper explores the relationship between early LRFD terms and discusses how today's differing terminology interpretations can lead to confusion. The second paper discusses several assumptions underlying all reliability analyses, and clarifies how these assumptions lead to end-use simplification (rather than complication). The third paper provides step-by-step calculations for users to follow when developing input parameters for reliability analysis. Finally, the fourth paper traces the history of closed-form reliability equations from the late 1960s to today's ASCE 7 recommendations.

In addition to the co-authors, many people contributed to these articles. We would like to acknowledge the comments received from Conroy Lum, Jeff Linville, Bob Tichy, B.J. Yeh, David Rosowsky, and John van de Lindt. On behalf of the many authors that contributed to this issue, we hope you find this information useful. Your comments and questions are welcome.

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Keeping Pace with Evolving LRFD Terminology in Design Standards

By David S. Gromala, PE, Philip Line, PE, Joseph F. Murphy, PhD, Thang Dao, PhD

Background

Textbooks and most major references on the subject of reliability analysis and reliability-based design – the basis of today's strength design provisions (also called load and resistance factor design, LRFD) – commonly use the term "nominal" to refer to specific values of loads and resistances. While each individual reference appears to use the term in a consistent manner, inconsistencies begin to appear when one compares specific reliability analysis inputs and outputs between different references over time.

To provide context, the generic design equation for LRFD is expressed as:

Factored nominal resistance ≥ Total factored nominal load

or

$\phi R_n \geq \Sigma \gamma_i Q_{ni}$

where:

ϕ = resistance factor

R_n = nominal resistance

γ_i = load factor(s)

Q_{ni} = nominal load(s)

The primary purpose of this article is to examine the manner in which nominal values have been defined in various reference documents and to propose a standardized definition – and calculation method – for the term "nominal resistance" for the design of wood structures using LRFD procedures (ASTM, 2015; AWC, 2015). Reference documents of interest range from those used during development of wood LRFD in the 1980s to modern editions of the *International Building Code* (ICC, 2015) and *ASCE 7 Minimum Design Loads for Buildings and Other Structures* (ASCE, 2010).

A secondary purpose of this article is to clarify another potential area of confusion related to wood LRFD terminology. The primary code (IBC) and load (ASCE 7) standards relevant to LRFD of wood structures, use the term "nominal strength" to describe the strength of a member before application of strength reduction factors. However, the *National Design Specification® (NDS®) for Wood Construction* uses a different term – "LRFD reference resistance." These terms can be used interchangeably.

KEYWORDS : nominal resistance, design resistance, characteristic value, wood structural element, NDS

Overview of Sources of Inconsistency

Before examining sources of inconsistency in terminology, several basic terms require definition, and underlying assumptions and specific areas of evolution of reliability methods and design standards warrant discussion.

Nominal

Dictionary definitions of the word "nominal" include:

- relating to, or being a noun or a word or expression taking a noun construction
- relating to, or constituting a name, or bearing the name of a person
- existing or being something in name or form only (i.e., *"nominal head of his party"*)
- being, or relating to a designated or theoretical size that may vary from the actual (approximate) (i.e., *the pipe's nominal size*)

– trifling, insignificant (i.e., *his involvement was nominal; charged only nominal rent*)
– a *rate of interest* equal to the annual rate of simple interest that would obtain if interest were not compounded when in fact it is compounded and paid for periods of less than a year or equal to the percentage by which a repaid loan exceeds the principal borrowed with no adjustment made for inflation
– being according to plan; satisfactory (i.e., *everything was nominal during the launch*)

While some of these definitions are obviously irrelevant when discussing structural reliability, they share a common trait – they are all relatively vague in their meaning. In fact, one could debate which definition would be most appropriate to cover the concept of a "nominal design value."

In wood research, design, and construction the confusion is compounded by the other common uses of the term:
– The most common context for this word in wood products is the term "nominal size" – such as "nominal dimensions of a 2x8." For example, the nominal dimensions are 2" thickness and 8" width for dressed dimension lumber that is actually 1.5" thick and 7.25" wide.
– Research reports in this field sometimes define the population 5th percentile value as the "nominal value," while others equate the allowable stress design value with the "nominal design value."

During the development of ASTM D5457 in the late 1980s, it was common in wood reliability studies to establish the mean/nominal ratio by computing the mean/5th percentile of a 2-parameter Weibull distribution with a specified COV. Many have asked: "Why don't we continue to use the 5th percentile value as the nominal value?" While there are some compelling reasons to retain this historical approach, it conflicts with modern codes and standards requirements and terminology for strength design used by engineers and other building design professionals.

Strength

Today's IBC and ASCE 7 terminology for strength design include the terms "nominal strength", "design strength" and "resistance factor." IBC definitions follow:

> ***Strength Design***. *A method of proportioning structural members such that the computed forces produced in the members by factored loads do not exceed the member design strength [also called "load and resistance factor design" (LRFD)]…..*
>
> ***Design Strength.*** *The product of the nominal strength and a resistance factor (or strength reduction factor).*
>
> ***Nominal Strength***. *The capacity of a structure or member to resist the effects of loads, as determined by computations using specified material strengths and dimensions and equations derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions.*
>
> ***Resistance Factor***. *A factor that accounts for deviations of the actual strength from the nominal strength and the manner and consequences of failure (also called "strength reduction factor").*

While these terms have been in effect for some time in the IBC and ASCE 7, early implementation of LRFD (Ellingwood, et. al., 1980; Rackwitz and Fiessler; 1978; Thoft-Cristensen and Baker, 1982) focused on the philosophical difference between allowable stress design (also called "working stress design") and the new LRFD procedures (called "strength design"). A key point in these discussions was the belief that structural analysis of behavior at or near the strength limit state was superior to analysis at lower stress levels. The differences between these two approaches can be significant when the structural elements exhibit nonlinear behavior at higher stress levels. Conversely, the two approaches are equivalent for linear-elastic behavior.

In today's building codes (ICC, 2015) and standards (ASCE, 2010), the term "strength" (rather than "stress") is used universally for LRFD. For example, terms "strength" and "stress" are used in part to differentiate between two primary design procedures in ASCE 7 as follows:

> *1.3.1.1* ***Strength Procedures****. Structural and nonstructural components and their connections shall have adequate strength to resist the applicable load combinations of Section 2.3 of this standard without exceeding the applicable strength limit states for the materials of construction.*
>
> *1.3.1.2* ***Allowable Stress Procedures****. Structural and nonstructural components and their connections shall have adequate strength to resist the applicable load combinations of Section 2.4 of this standard without exceeding the applicable allowable stresses for the materials of construction.*

Conceptually, the strength of a structural element is its moment capacity (beams), axial capacity (tension or compression), etc. Conversely, engineers designing buildings using wood products typically compute design strength by multiplying an LRFD value (i.e. stress value) by a section property. From a practical perspective, the strength design limit state can be expressed in terms of stress or strength to produce the same element design size. When ASCE 7 uses the term "nominal strength," it is equivalent on a stress value basis to the NDS term "LRFD reference resistance". The designer can compute the member nominal strength on a member capacity basis by multiplying the LRFD reference resistance value (i.e. stress value) times the appropriate section property.

Historical Perspective

If one considers the National Bureau of Standards Special Publication 577 (NBS SP577, Ellingwood, et. al., 1980) to be the primary historical reference, the evolution of the term "nominal" becomes a bit more understandable. This publication used the term "nominal" in very broad context given the fact that at the time there were no code-prescribed values for LRFD (either for loads or for resistances). The appendix provides more details from NBS SP577, enabling readers to infer its intent.

It is important to note that the term "nominal resistance" from these 1980s-era references is similar, but not identical, to its typical definition in today's standards.

Defining Terms for Strength Design (LRFD) for Wood Products

For consistency with evolving reliability provisions in codes and standards, the following terminology is proposed for use in reliability analysis of wood products:

Nominal resistance (R_n): A term used in reliability standards equivalent to the reference resistance.

Reference resistance (R_n): The design value used in LRFD equations to represent member resistance prior to application of the resistance factor (ϕ) and adjustment for end-use conditions. (Also known as "LRFD reference resistance" in the NDS.)

Design resistance: The product of reference resistance (adjusted to end-use conditions), the resistance factor, and the time-effect factor.

Because published LRFD values for many wood products are related to previously-derived allowable stress design (ASD) values, it is useful to add the following term:

Characteristic value: The value used as the basis (typically test-based) for establishment of design values. For many wood products, the population estimate of the fifth-percentile value, $R_{.05}$, is typically used.

Example Derivation of Nominal Resistance

The NDS (AWC, 2015), referencing ASTM D5457 (ASTM, 2015), establishes design values for LRFD based on a format conversion formula tied to allowable stress design (ASD) values. For most wood member strength properties, allowable stress design values are based on test-based population estimates of the 5th percentile of the

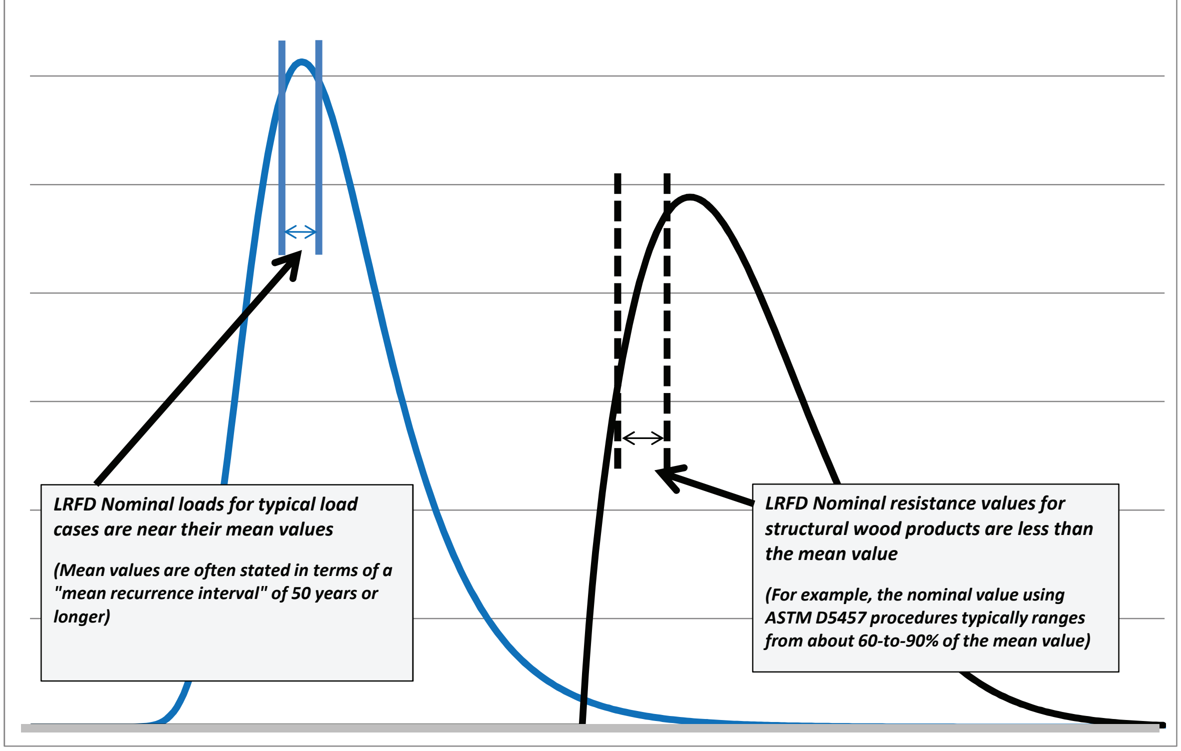

Figure 1. Illustration of Typical Range of LRFD Nominal Load (Q_n) and LRFD Nominal Resistance (R_n) Relative to Distributions

resistance distribution. The calculation process proceeds as follows:

R_n = the LRFD reference resistance (or nominal resistance)

R_{ASD} = the reference design value for use in ASD

$R_{.05}$ = the population 5th percentile estimate used as the basis for establishing design values

In accordance with NDS and ASTM D5457, these three terms are related in the following equation:

R_n = (Format conversion factor, K_F) x R_{ASD}

where:
Format conversion factor, $K_F = 2.16/\phi$ (defined in ASTM D5457)

$R_{ASD} = R_{.05}/2.1$ (where 2.1 is the typical reduction factor per the applicable product standard)

For example, compute the LRFD nominal resistance for a 2400f glulam roof beam (dead+snow loading):

R_{ASD} = 2400 psi

ϕ (bending) = 0.85

Format conversion factor (K_F) = 2.16/0.85

R_n = (2.16 / 0.85) x 2400 = 6098 psi

Using the terminology proposed herein (assuming no additional end-use adjustments are needed) LRFD nominal resistance and LRFD design resistance are as follows:

Nominal resistance (R_n) = 6098 psi

Design resistance: $\lambda\phi R_n$ = 0.80 x 0.85 x 6098 = 4147 psi

Figure 1 illustrates the typical range of LRFD nominal values for loads (ASCE 7-10 basis) and resistance (ASTM D5457 format conversion basis).

Summary

This article has provided background information related to the historical evolution of several key terms used in reliability analysis and in wood LRFD provisions. The updated terminology related to LRFD nominal resistance for wood design reflects present day use of nominal strength in association with strength design methods in modern codes and standards.

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Appendix: NBS SP577 Language Related to Nominal Resistance

When establishing input parameters for the reliability analysis in NBS SP577, all of the nominal values (both loads and resistances) in the design checking equation were non-dimensionalized by dividing all nominal values by the dead load nominal. The dead load nominal was chosen as the baseline because the dead load is included in all of the checking equations. For each load combination containing additional load variables, each was normalized as a ratio to the dead load (for example, L/D ratios might be analyzed from a range of 1.0 to 9.0).

Similarly, resistance parameters were non-dimensionalized (i.e., R_n/D_n). Thus, proper definition of the "nominal value" for the resistance variable is critical to the entire analysis. Unfortunately, guidance within NBS SP577 is scarce regarding how to establish the appropriate nominal value for this analysis.

For example, page 2 of NBS SP577 states that: "The term R_n is a nominal resistance corresponding to a limit state (e.g., maximum moment which can be carried by a cross section, buckling load, shear capacity), and ϕ is the "resistance factor" which is less than unity and reflects the degree of uncertainty associated with the determination of the resistance."

Moving to the top of page 22 of NBS SP577, an interesting clarification regarding the term "nominal value" is provided: "…..in which $X_{n,i}$ is the nominal or design value of the load or resistance parameter specified in the building standard."

As further clarification of the intent in defining the nominal, page 48 of NBS SP577 portrays R_n as FS x (D_n+L_n+W_n) x (3/4), where FS = factor of safety. This definition is consistent with the relationship between the allowable stress under this load condition and the specified yield stress for steel design in 1980.

The notation in the heading at the top of Table C.7.1 on page 158 of NBS SP577 reinforces the previous point by including additional information. The heading states that $R_m/R_n = 1.05$. This indicates that the mean is 1.05 times the "design value specified in the building standard" (to borrow from Page 22 wording). Since the heading also specifies FS = 5/3, it is clear that NBS SP577 in this example was evaluating the reliability of the then-current allowable stress design for these various loading conditions.

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Do Design Adjustment Factors for Wood Affect the Computed Reliability Index: And If Not, Why Not?

By David S. Gromala, PE, Philip Line, PE, Joseph F. Murphy, PhD, Kevin C.K. Cheung, PhD

Overview

There are many adjustment factors applied in the design of wood structural elements to account for conditions that vary from "reference conditions." It is important to note that adjustment factors are different from safety factors. Design adjustment factors, such as for wet service or temperature, are appropriately considered in reliability analysis when both the resistance distribution and the design checking equation are "adjusted" in a consistent manner. The assumption underlying most wood reliability analyses is that the designer-applied adjustment factor within the design checking equation precisely matches the adjustment in the resistance distribution and that all points in the distribution are scaled by the same amount.

Many technical articles related to reliability analysis present their input parameters in property ratios or other non-dimensional terms. Reliability analyses presented in this manner are often difficult to relate to real design applications and are often unclear as to whether design adjustment factors are part of the reliability index calculation. In this paper, the effect of design adjustment factors is examined using actual design examples with actual design values and design spans to illustrate how design adjustment factors affect the computed reliability index.

Background

The early developers of reliability analysis – and especially those charged with implementing it in a user-friendly format – wrestled with a fundamental question: What sources of variability should they consider in the analysis? One can postulate dozens, if not hundreds, of individual sources of variability that will impact the final performance of a building. These sources of variability include, among others, single member versus system level response of the structural configuration being evaluated for a given limit state, accuracy of design analog versus actual state of stress in the member, accuracy of design adjustment factors across ranges of grades and member sizes, and interaction of all the above. On a practical level, it is impossible to quantify whether each source of variability applies equally across the entire resistance population (i.e. range of member size, grade, orientation and use within the structural system). Additionally, when one contemplates whether various factors might be correlated – where one factor might influence another – the problem becomes intractable.

Nearly without exception, those implementing LRFD in various codes and standards have chosen to exclude certain sources of variability from the reliability analysis by use of simplifying assumptions. These simplifying assumptions include analysis of single members without explicitly accounting for system level response -- which might include both positive effects (i.e., the beneficial effects of load sharing between adjacent repetitive framing members and composite action of repetitive member sheathed assemblies) and negative effects (i.e., overall system size effects which suggests an increasing probability of having a weak link failure as the number of structural members increases).

Embedded within virtually all reliability analyses is the assumption that any design adjustments to reflect

KEYWORDS : adjusted resistance, time effect factor, LRFD, NDS

end-use conditions are 100% accurate. In other words, the implied assumption is that a factor of 0.9 applied in the design will scale the entire resistance distribution precisely by a factor of 0.9. Additionally, it is assumed that the combined effect of various design adjustments in accordance with the design checking equation has the same effect on the resistance distribution.

Fundamental Assumption Underlying All Reliability Analyses

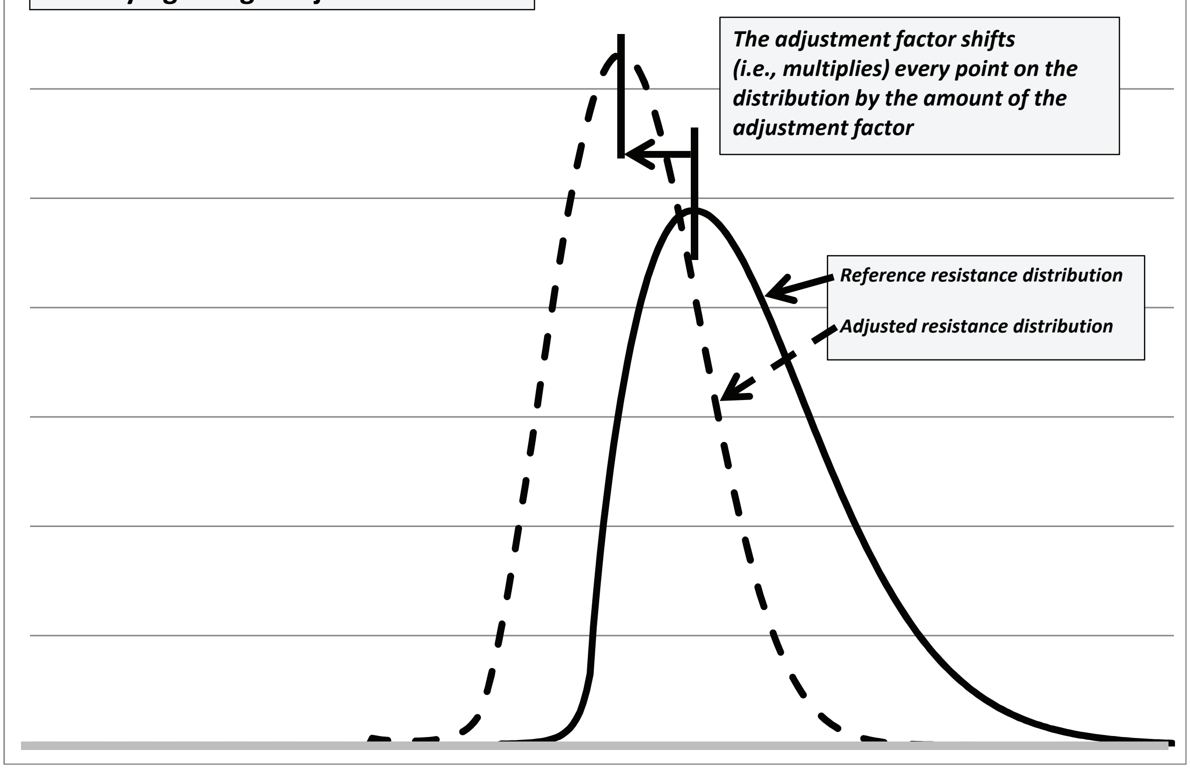
To reiterate this important fact - a significant, and underappreciated, fundamental assumption underlying all reliability analyses is that all design adjustment factors using the *National Design Specification®* (*NDS®*) *for Wood Construction* (*AWC*, 2015) - those applied by the designer to adjust to specific environmental or other end-use conditions – "scales" (i.e., multiplies each value of) the entire population by exactly the amount of the adjustment factor (Figure 1). Some might propose that this assumption is too simplistic, and others might propose research studies to prove a more complex situation. However, when one examines the nearly infinite combination of product types, grades, sizes, stress modes, and environmental conditions, one can see that no research study could be sufficiently comprehensive to justify changing this fundamental assumption.

Generic Example: Adjustment Factors for Lumber Bending

The list of adjustment factors that could conceivably be applied to the reference design value for lumber can be extensive (i.e., $\lambda \bullet C_r \bullet C_F \bullet C_L \bullet C_M \bullet C_{fu} \bullet C_P \bullet C_t \bullet C_i \bullet C_b \bullet C_f$). Since this is for illustration only, the terms in the list will not be defined here.

All of these factors are assumed to be independent of one another. Most of these factors were determined in various test programs comparing a control group and an "adjustment" group (typically called a "treatment" group in research studies). While these adjustment factors were determined based on differences between the means of the data sets, each factor has been assumed

Figure 1. Fundamental Assumption Underlying Design Adjustment Factors



to apply to the entire property distribution. Additionally, it is important to note that each factor is applied as a constant rather than as a random variable.

In equation form, the adjusted nominal resistance equals the unadjusted (i.e., reference) resistance multiplied by the aforementioned list of factors. And, since each factor applies equally to the entire property distribution, the adjusted mean resistance similarly equals the unadjusted (i.e., reference) mean multiplied by the same list of factors, which leads to the following conclusions:

- The ratio of adjusted mean-to-nominal is identical to the ratio of the "reference" mean-to-nominal.
- The coefficient of variation of the adjusted distribution is the same as that of the reference distribution.

Practical Design Example

To provide context, the following examples are provided in both allowable stress design (ASD) format (also called "working stress design") and load and resistance factor design (LRFD) format (also called "strength design").

Start with a tested population of 2400f glulam beams with mean modulus of rupture of 7608 psi (R_M = 7608 psi) and 17.2% COV (V_R = 17.2%).

Check the design of a 5-1/8 in. x 12 in., 2400f glulam beam.

Application: Roof beam, 18-ft - 9-in. span, 8-ft spacing, 20 psf dead load, 60 psf snow load

Allowable Stress Design: Resistive moment > Applied moment

Design checking equation (all terms expressed in correct units): $C_D \times F_b \times S > D + S$

Resistive moment (ASD): 1.15 (C_D) x 2400 psi x S (section modulus) = 339480 in.-lb.

Applied moment (unfactored loads): 640 plf x $(18.75)^2$ / 8 x 12 (in./ft.)= 337500 in.-lb.

<u>99.4% of design capacity</u>

Load and Resistance Factor Design (LRFD): Resistive moment > Applied moment

Design checking equation (all terms expressed in correct units): $\lambda \times \phi \times R_n > 1.2 \times D + 1.6 \times S$

Nominal Design Value (R_n): 2.16/ϕ x 2400 = 6098 psi

Resistive moment (LRFD): 0.80 (λ) x 0.85 (ϕ) x 6098 psi x S (section modulus) = 510105 in.-lb.

Applied moment (factored loads): [1.2 x 160 plf (dead) + 1.6 x 480 plf (snow)] x $(18.75)^2$ / 8 x 12 (in./ft.) = 506250 in.-lb.

<u>99.2% of design capacity</u>

Conclusion #1. As expected, ASD and LRFD produce the same designs due to the nature of format conversion implemented in *NDS*, for design of wood construction. This example has been provided simply to remind the reader of the steps followed in design for ASD and for LRFD.

Next, Compute the Reliability Index (LRFD)

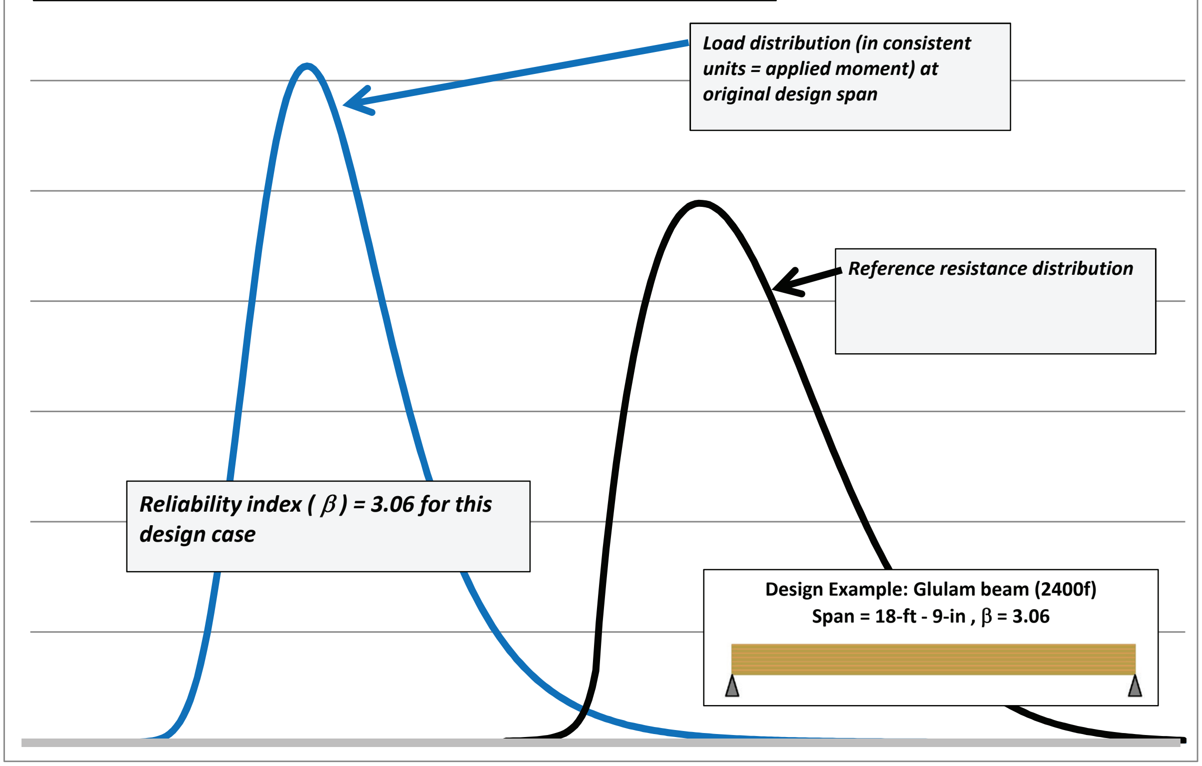
Reliability calculations for this example will use closed-form equations for simplicity and input variables as defined by Gromala, et al., 2017. Figure 2 illustrates the relationship between the load distribution (expressed as the applied moment) and the resistance (expressed as the resistive moment).

R_M/R_n = 7608 / 6098 = 1.247

V_R = 17.2%

R_n/D_n = 7.059

Figure 2. The Reliability Index (β) is Computed Based on the Probability of the Load Exceeding the Resistance



$Q_M/D_n = 4.05$

$V_Q = 18.7\%$

$R_M/Q_M = (R_M/R_n) \times (R_n/D_n) / (Q_M/D_n) = 2.174$

Reliability Index, $\beta = 3.06$

Modified Design Example: Beam application is (unexpectedly) a wet-use condition

Using the same assumptions as in the previous example, assume that the application is subject to high moisture conditions such that a wet service adjustment factor, C_M, of 0.80 should have been applied (but was somehow not addressed in the design phase).

The population mean is reduced by the wet service factor:

$R_M(\text{adjusted}) = 7608 \times 0.80 = 6086$ psi

Compute the reliability index under the assumption that the designer does not apply the wet service factor and retains the 18-ft - 9-in. span. Since the mean under wet-use conditions has decreased to 6086 psi but the designer has not adjusted the nominal resistance, the ratio of R_M/R_n has decreased to 0.998 and the reliability index is now 2.18. Figure 3 depicts this (flawed) design case.

Correctly Adjusted Design Example

Next, assume that the beam was intended for a wet-use application and that designer correctly applies the 0.80 wet service factor to the design. The adjusted nominal resistance is 4869 psi, the span is calculated as 16-ft - 9-in., the ratio of R_M/R_n remains at 1.247 (same as the base case), and the computed reliability index remains at 3.06 (same as the base case). As illustrated in Figure 4, while the resistance population has indeed been scaled, the designer has accommodated this adjustment by reducing the span (and therefore, the applied moment) by the same amount – retaining the reliability of the original design.

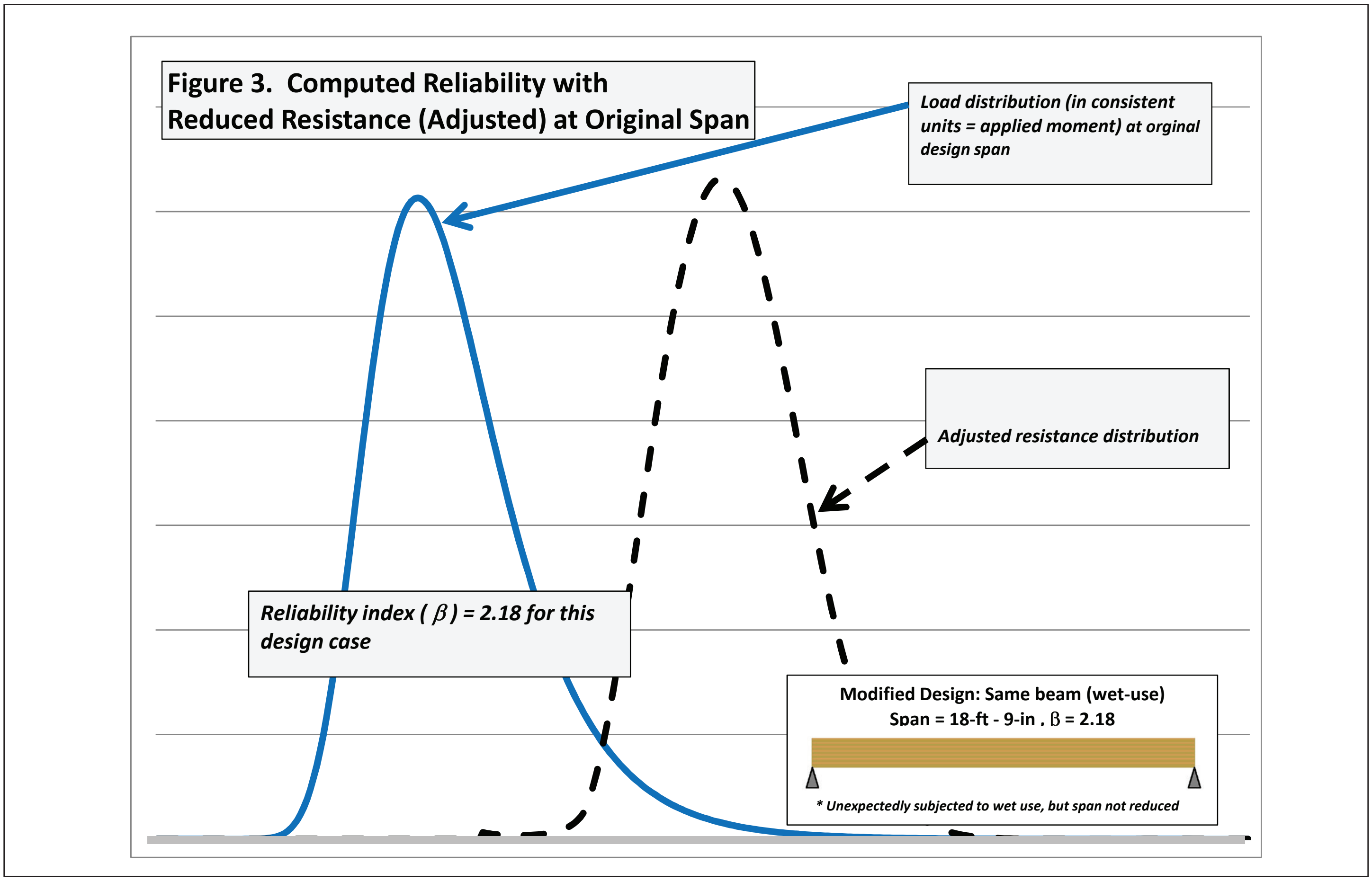


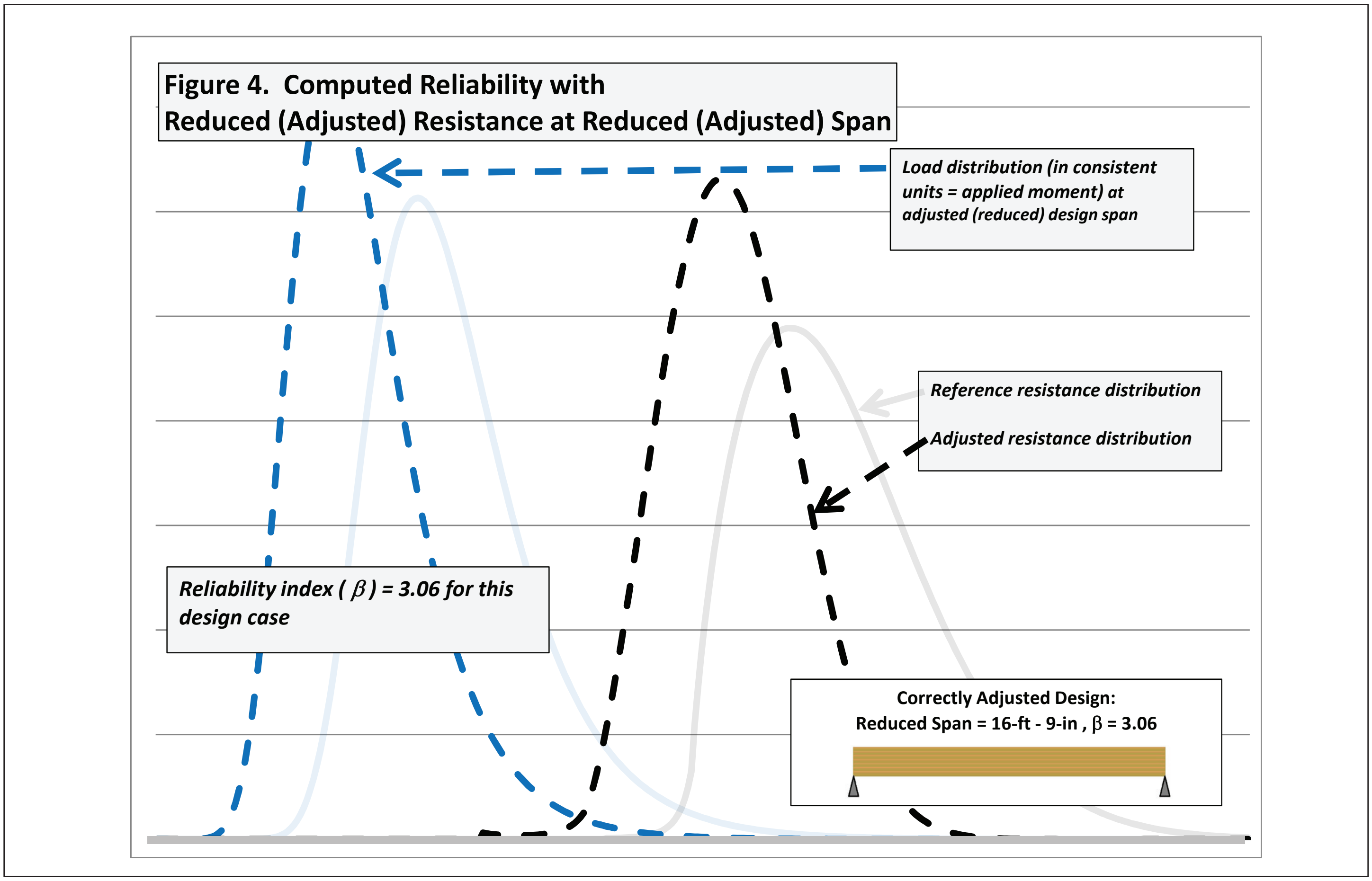
Figure 3. Computed Reliability with Reduced Resistance (Adjusted) at Original Span

The Special Case of Time Effect Factors

If one applies the same logic (i.e., where the population resistance reduction equals the reduction in applied load effect), to the time effect factor, this factor should also be omitted from the reliability analysis. Implicit in this concept is that when the designer applies, for example, a 0.80 time effect factor for a snow load design, the resistance population (under the lifetime load history) would be expected to scale by the same factor.

Examine, for example, what would happen if one applies a different perspective. What if the time effect factor is applied in the design checking equation (i.e., the adjustment included in the design), but the resistance statistics remain unchanged from the reference case? Because the resistance would be unchanged while the applied load effect is reduced, the time effect factor would enter the analysis as a "bonus factor of safety" applied above and beyond the standard factors. This is precisely the situation in the reliability indices shown in Figures 3 through 7 of the reference paper by Rosowsky, et al. (2005). This paper was being developed while various wood LRFD implementation decisions were still in a state of flux, and the paper attempted to cover a broad range of high-profile topics. For example, the paper is still the single best source of consolidated load distribution information. It was one of the first to attempt to characterize various classes of structural wood products using "benchmark" values for mean-to-nominal ratios and coefficients of variation. It was one of the first to address the stochastic basis of the *NDS* time effect factors and compare that basis against the scalar adjustment factor approach. However, since the publication of the 2005 paper, its computed range of reliability results (roughly 2.8 to 3.4 for most load combinations) is inconsistent with other studies for which the time effect factor is applied to both the resistance distribution and the design checking equation.

This anomaly can be explained by closer examination



of the 2005 paper, which states: *"Note that Eq. 2 does not take into account load duration (time) effects. It therefore only considers "overload" type failures, or failures in which the stress caused by the highest load (or combination of concurrent loads) exceeds the short-term strength."* The paper continues with: *"To simplify the analyses in this study, it is assumed that the time effects factors in the current ASCE 16 standard are properly calibrated and therefore Eq. 2 is used for the limit states analyses. It is further assumed that all adjustment factors (C_i) properly take into account their respective effects on resistance. In this analysis, the factors default to unity……"*

One additional note regarding the time effect factor (λ) is worth discussing. As discussed herein, the Rosowsky, et al. (2005) analysis effectively treats λ as an additional safety factor for D+L and D+S cases (yielding an upper bound on the reliability estimate) – because it assumes no scaling of the distribution, but accepts a designer-applied reduction factor. A lower bound on the reliability estimate would coincide with the time effect factor being similar to other adjustment factors – where the distribution scaling is assumed to be identical to the designer-applied factor. Research studies, as discussed by Karacabeyli and Soltis (1991), have shown that loads applied over extended periods affect lower strength pieces more than the rest of the distribution - which differs from other design adjustment factors. The best estimate of this strength reduction is reflected in the time effect factors in the *NDS*. Thus, the actual impact of duration of load on the computed reliability is somewhere between these two bounds. Users should note that this conceptual difference is unique to the time effect factor, and that it is only relevant in the calculation of the reliability index – it does not affect design of wood structural elements.

Summary

On a practical level, it is impossible to quantify whether every design adjustment factor applies equally across the entire resistance population or whether one or more concurrent adjustments might be correlated. The *NDS*, and similar structural material specifications in the U.S. apply design adjustment factors in a scalar (i.e., multiplicative) manner without adding to the resistance variability. Thus, embedded within virtually all reliability analyses are two implicit assumptions (discussed in more detail in the Appendix):

1) All design adjustments scale the entire distribution equally (i.e., the mean scales by the same factor as the nominal), thus preserving the mean-to-nominal ratio unchanged; and

2) All design adjustments to reflect end-use conditions are 100% accurate (i.e., the resistance COV remains the same before and after scaling).

Thus, when examined in the context of the complete design process – where designers account for the adjustment by changing (typically reducing) the applied load effect (either by increasing member grade/size or by reducing the span) – it can be shown that design adjustment factors have no impact on the computed reliability index.

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Appendix: Multiplying a Random Variable by a Constant and Discussion of its Effect on the Population

The math describing the application of an adjustment factor can be expressed in terms of the multiplication of a random variable by a constant, as follows:

For a random variable, X, the expected value (i.e., mean), the variance (Var), and the coefficient of variation (COV) of X can be expressed as:

Expected value = $E(X)$

Variance = $Var(X)$

Coefficient of variation (COV) = $\frac{\sqrt{Var(X)}}{E(X)}$

If C is a constant, then:

The expected value of $C \bullet X = E(C \bullet X) = C \bullet E(X)$

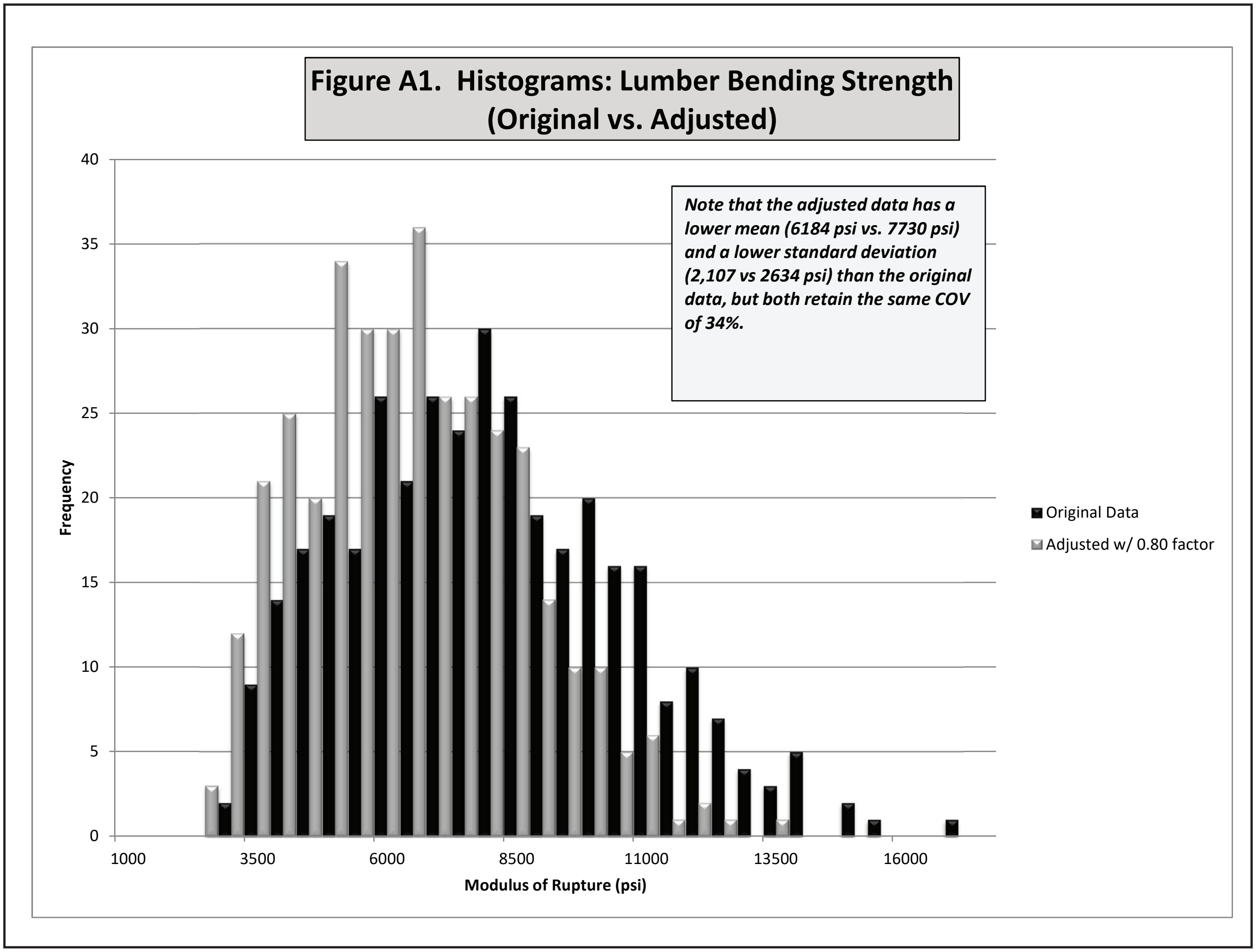
The variance of $C \bullet X = Var(C \bullet X) = C^2 \bullet Var(X)$

The COV of $C \bullet X = COV(C \bullet X) = \frac{\sqrt{Var(C \bullet X)}}{E(C \bullet X)} = \frac{\sqrt{C^2 Var(X)}}{C \bullet E(X)} = COV(X)$

Therefore, multiplying a random variable distribution by a constant does not change its coefficient of variation.

The same logic confirms that the ratio of the expected value (mean) to the nominal value is the same before and after application of the constant.

Figure A1 shows histograms for a data set of lumber (bending) before and after adjustment by a 0.80 design adjustment factor, confirming the concepts described above.



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Example Calculations to Derive Input Values for Reliability Analysis of Wood Products

By David S. Gromala, PE, Philip Line, PE, Joseph F. Murphy, PhD, Thang Dao, PhD

Background

Structural reliability analysis has evolved in many ways over the past four decades. However, some of its basic features and underlying assumptions have remained surprisingly stable. Unfortunately, because many facets of reliability analysis involve relatively complex mathematical algorithms, casual users are often confused by differing nomenclature and methodologies.

The purposes of this article are to review several key references, to identify subtle differences in their approaches, and to propose a common framework for future use of reliability analyses for wood products.

Structural Reliability Analysis: A side note that provides some context

The underpinnings of structural reliability analysis have been well-documented in the literature. Hasofer and Lind (1974) are often referenced regarding the basic methodology. Galambos and Ravindra (1973) published many articles proposing techniques for implementing reliability concepts in a material design specification. Rackwitz and Fiessler (1978) are typically mentioned when discussing extended procedures that include non-normal analysis variables. However, it is the procedures from the National Bureau of Standards Special Publication 577 (Ellingwood, et. al., 1980) that formed the basis of commonly-used reliability analysis procedures and of the load combinations in today's national load standard in the U.S. – ASCE 7 *Minimum Design Loads for Buildings and Other Structures* (ASCE, 2010). NBS SP577 provides complete calculation algorithms and FORTRAN-based software code for structural reliability analysis, and is the template for the input values discussed in this article.

In its simplest conceptual form, reliability analysis uses one probability distribution (the "resistance") and another probability distribution (the "demand") and computes the probability (or frequency of the number of times) that the demand exceeds the resistance (i.e., exceeds the "limit state"). Every reliability analysis, regardless of the number of random variables and the complexity of the algorithm, is intended to perform this basic calculation.

The Hasofer-Lind approach solved the equations in closed-form to compute a reliability index, β. Their examples include cases with one resistance distribution and one demand (i.e., load) distribution, but their approach could be extended to include multiple random variables. The appeal of this approach is that it is both mathematically exact and intuitively easy to understand. Because the closed-form equations only require users to input the mean and coefficient of variation (abbreviated herein as COV), the reliability index does not necessarily imply a specific probability of failure unless the random variables are normally distributed.

The Rackwitz-Fiessler approach extended the analysis by permitting additional variables and by accommodating other distribution forms. It accomplishes these tasks using an iterative approach that seeks an optimal point at which it transforms each variable's distribution into an equivalent normal (i.e., having the same cumulative distribution function, CDF, and probability density function, PDF, at that point) to compute the reliability index. This is the approach used in NBS SP577.

It is important to note that reliability analysis concepts were

KEYWORDS : LRFD, input parameters, reliability software, FOSM, FORM

being introduced at approximately the same time that the structural engineering community was contemplating the move from allowable stress design (ASD, also called "working stress design") to load and resistance factor design (LRFD, also often called "strength design"). There are distinct advantages to analyzing structural behavior at an elevated state of stress (in the vicinity of its strength limit state) when subjected to loads at or near their expected maximum lifetime levels – especially for conditions where nonlinearity or other load redistribution might occur. Conversely, for other structural applications, where response is reasonably linear-elastic to failure, the differences between ASD and LRFD are more subtle (and are primarily a transfer of a portion of so-called "safety factors" from the resistance side to the load side of the design equation). Thus, while designers see larger numbers (such as resistive moments and applied moments) in LRFD than in ASD, the underlying design case is relatively unchanged.

Scope of this paper

This paper provides the arithmetic required to generate consistent input values for reliability analysis. To provide context, reliability analysis requires definition of the random variables of interest (typically the loads and the resistance – expressed in consistent units, such as moment or axial force) for the specific case defined by the design checking equation.

Calculation Steps

NBS SP577 provides a Fortran-based software methodology for reliability analysis. Rather than using actual design quantities (such as stress or moment capacity), the software uses non-dimensional inputs (such as ratios of mean-to-nominal values, coefficients of variation, etc.). This provides a convenient way to examine a range of design cases that have broader applicability.

There are two parts to the input sequence in NBS SP577-style analyses:

1) Characterize the input distributions for loads and

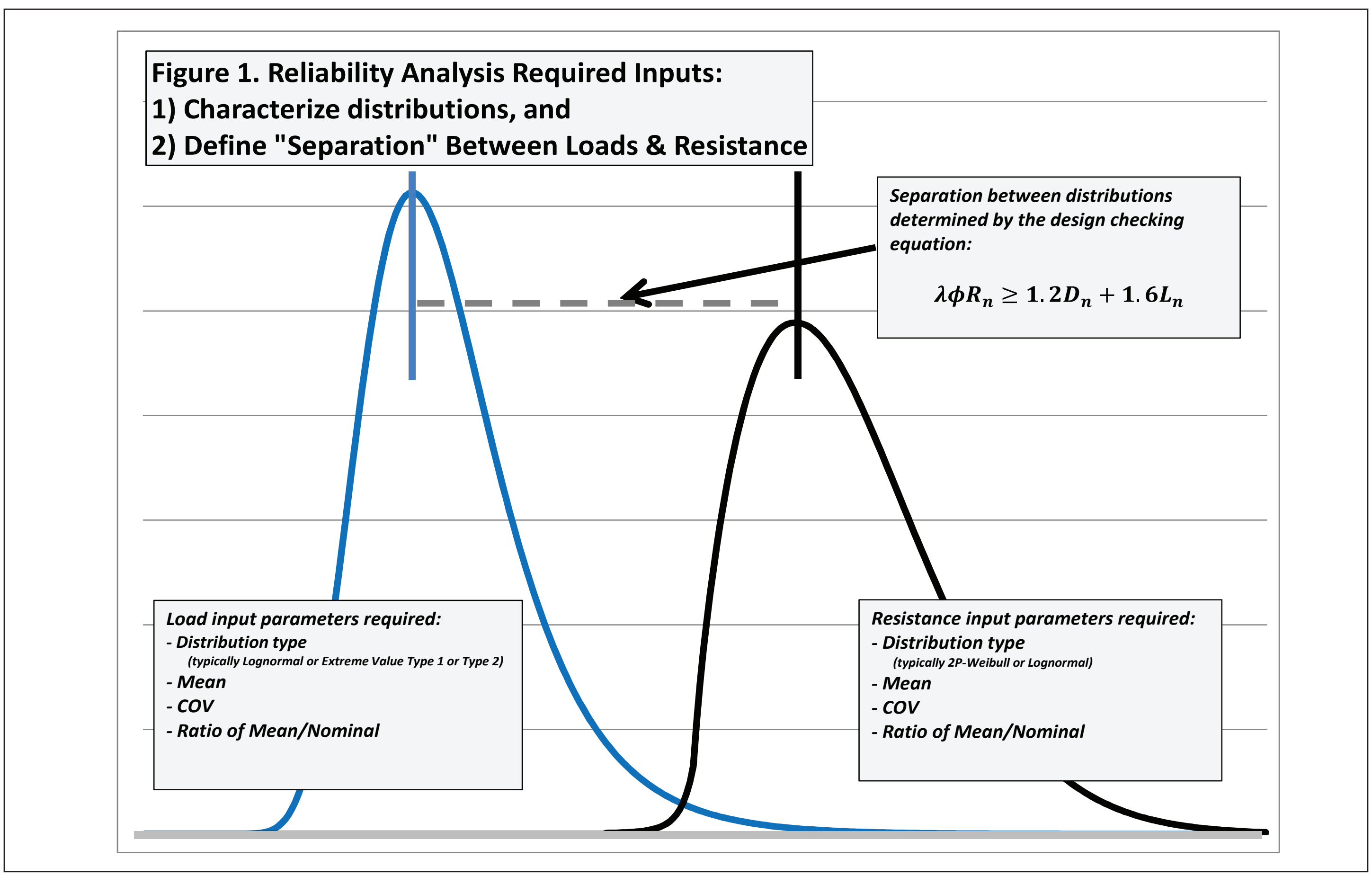

Figure 1. Reliability Analysis Required Inputs:
1) Characterize distributions, and
2) Define "Separation" Between Loads & Resistance

resistance (mean-to-nominal ratio, COV, distribution type)

2) Position the distributions relative to each other (nominal values expressed non-dimensionally by dividing each nominal by the dead load nominal)

These steps are visually displayed in Figure 1.

Discussion regarding input distributions

It is important to note that the inputs for each variable in the analysis **must** include the following:

1) The distribution type
   a. Commonly referenced distribution types for ASCE 7 load cases include normal, lognormal, Extreme Value Type 1 (also known as Gumbel), and Extreme Value Type 2 (also known as Freche)
   b. Commonly referenced distribution types for material resistances include normal, lognormal, and Weibull

2) The MEAN value (or mean-to-nominal ratio)
   a. Typically designated with the variable notation having a line or "bar" over it, or alternatively, with a subscript "M" (i.e., "D-BAR" or D_M)

3) The COEFFICIENT OF VARIATION
   a. Typically designated as COV or by V with subscript R or Q to designate resistance or load, respectively (i.e. V_R or V_Q)

4) The NOMINAL value (if the mean-to-nominal ratio is used)
   a. Typically designated with the variable notation and a subscript "n" (i.e., D_n)

Note that there are two common ways by which this information is input. In the NBS SP577 method, the initial inputs for each distribution are expressed in terms of mean-to-nominal ratios, COV's, and normalized nominal values (normalized to the nominal dead load). Conversely, other high-level mathematical software routines often input the mean, COV, and nominal separately for each distribution. Appendix A2 discusses this point in greater detail.

Discussion regarding "positioning" loads versus resistance

When all variables are translated into consistent units, the idea of "positioning the distributions" is easier to understand. In structural design, "loads" are not simply loads – they are load effects (for example, total applied moment for a bending member). Similarly, in a beam design, "resistance" is not simply resistance – it would be stated as "resistive moment." In practical terms, the designer computes the maximum span for a structural member (where the applied moment equals the resistive moment) – thus "positioning" the combined load distribution relative to the resistance distribution.

Characterizing the Loads

The input values for loads included in ASCE 7's load combinations include mean, COV, and distribution type and are available in reference standards and reliability literature. Rosowsky, et al. (2005) summarizes commonly-used statistics and provides a more comprehensive set of load cases than most other references – including regional statistics for snow and for wind. These values are duplicated in Table 1 for convenience.

A key point when using NBS SP577 style of software is to understand that each variable is initially expressed in terms of its mean-to-nominal ratio, after which the nominal value is input in terms of its ratio to the nominal dead load (i.e., input as R_n/D_n).

Example: For a design with a 10 psf dead load and a 30 psf snow load, the dead and snow statistics for mean/nominal and COV are first input as shown in Table 1 (1.05 and 0.10 for dead; 0.82 and 0.26 for snow), and then the nominal values are input – with the nominal dead load input (D_n) as unity (1.0) and the nominal snow load input (S_n) as 3.0 (because S/D = 3).

Characterizing the Resistance

The wood products industry has a long history of conducting extensive test programs to characterize

Table 1. Load Statistics (from Rosowsky, et al., 2005)

Load	Footnote	Mean-to-Nominal Ratio	Coefficient of variation	Distribution
Dead		1.05	0.10	Normal
Live	a	1.00	0.25	Extreme Type 1
Snow	b	0.82	0.26	Extreme Type 2
Snow R1	c	0.61	0.53	Lognormal
Snow R2	d	0.84	0.60	Lognormal
Snow R3	e	0.80	0.58	Lognormal
Wind	f	0.78	0.37	Extreme Type 1
Wind R1	g	0.95	0.31	Extreme Type 1
Wind R2	h	0.68	0.29	Extreme Type 1
Wind R3	i	0.69	0.92	Extreme Type 2

[a] 50-year maximum total (sustained + extraordinary) occupancy live load.

[b] 50-year maximum snow load based on an aggregation of eight sites in northern tier states.

[c] 50-year maximum snow load based on new analyses (Northern tier sites).

[d] 50-year maximum snow load based on new analyses (Midwest / Mid-Atlantic sites).

[e] 50-year maximum snow load based on new analyses (Mountain West / Northwest sites).

[f] 50-year maximum wind load based on an aggregation of seven inland sites.

[g] 50-year maximum wind load for Des Moines, Iowa based on new analysis.

[h] 50-year maximum wind load for Denver, Colo. based on new analysis.

[i] 50-year maximum wind load for Miami, Fla. based on new analysis.

resistance distributions for structural wood products. The results from some of these programs form the basis of benchmark reliability analyses for wood products. However, since each test program and its summary reporting will include its own set of reference conditions, assumptions, and definition of terms, readers are cautioned to review each of these details when attempting to use them in reliability analyses.

Techniques for selection of the best distribution type to fit an individual data set have been reported in numerous references over many decades. For extremely large data sets, non-parametric statistics (rather than parametric) are often used. For purposes of reliability analysis, resistance distributions are typically characterized as lognormal or 2-parameter Weibull (although some references assume a normal distribution). Note that there is no inherent limitation in the reliability analysis regarding the choice of distribution.

As indicated previously for loads, each resistance variable used in the reliability analysis must be listed with a distribution type, mean (or mean-to-nominal ratio), COV, and nominal value. The first three items are self-explanatory. However, the term "nominal value" requires additional discussion.

The long history of wood products testing and reporting, and the continuing dominance of the allowable stress design method in the marketplace leads to a range of typical uses for the term "nominal" in wood design literature. Some references consider it to be the population 5th percentile. Other references use the word "nominal" and "design value" interchangeably.

However, for use in reliability analysis and for designing in LRFD, the term nominal resistance is defined as:

Nominal Value, R_n = The LRFD reference resistance value based on reference conditions before the application of any adjustment factors

As stated earlier, input values in NBS SP577-style analysis are first input as mean-to-nominal ratios, and subsequently as nominals normalized to the dead load. Thus, this part of the calculation requires two separate steps:

1) Compute the mean resistance, R_M, as a function of R_n ,
2) Compute R_n as a function of D_n (in consistent terms/units).

Computing R_M for wood products based on design value derivations

For some structural materials, determining the correct ratio of mean resistance (R_M) to nominal resistance (R_n) simply requires a standard reference book. However, because published design values for wood LRFD are based on format conversion from ASD, which in turn are typically based on product-specific test results, calculation of the ratio of mean resistance (R_M) to nominal resistance (R_n) ratio requires several steps.

In its generic form, the calculation of the mean-to-nominal ratio is:

$R_M/R_n = R_M$ (determined by testing) / R_n (specified LRFD reference resistance value)

When the mean resistance (R_M) is not determined by testing, it can be inferred based on several assumptions.

Table 2. Factors to convert the ASD value (R_{ASD}) to the nominal resistance (R_n)

$$R_n = K_F \times R_{ASD}$$

Application	Property	ϕ [1]	K_F [2]
Member	Compression Parallel to Grain	0.90	2.40
	Bending	0.85	2.54
	Tension Parallel	0.80	2.70
	Shear	0.75	2.88
	Radial Tension	0.75	2.88
	Lateral Buckling (Stability) [3]	0.85	1.76
	Compression Perpendicular to Grain [4]	0.90	1.67
Connection	all [5]	0.65	3.32
Shear Wall and Diaphragm Shear	all [6]	0.80	2.00

[1] Resistance factors as specified in Table 2 of ASTM D5457-15

[2] Format conversion factors as specified in Table 4 of ASTM D5457-15

[3] Lateral buckling is not subject to time effects

[4] Compression perpendicular to grain is not subject to time effects and is not 5th percentile based

[5] Connection design values are typically not 5th percentile based

[6] Shear Wall and Diaphragm Shear values are not 5th percentile based

If the population 5th percentile is assumed to be fixed at unity (1.0), R_M (or, more precisely $R_M/R_{.05}$), can be expressed across a range of COV for each assumed distribution type (as shown in Table A1.1).

The nominal resistance can similarly be computed as a function of population statistics. The examples shown herein are limited to products with ASD values based on the population 5th percentile and using ASTM D5457's format conversion factors to derive LRFD values. For these cases, the nominal resistance (R_n) is calculated by applying a factor, K_F, to the ASD value):

$$R_n = K_F \times R_{ASD}$$

As indicated earlier, the ratio of R_M/R_n can be determined directly using test data for the mean and the published (LRFD) design value for the nominal. However, if one wishes to get a sense of the range of likely ratios, there is another option.

In the early development of ASTM D5457 *Standard Specification for Computing Reference Resistance of Wood-Based Materials and Structural Connections for Load and Resistance Factor Design* (ASTM, 2015), reliability analyses were conducted across a range of hypothetical resistance values. The most common (conservative) assumption was that the population 5th percentile was exactly at the minimum permitted by the underlying product standard. This assumption, combined with an assumed standard distribution form, permits the tabulation of $R_M/R_{.05}$ across a range of COV's.

A range of $R_M/R_{.05}$ and R_M/R_n ratios are provided in Table A1.1 for lognormal, normal and 2-parameter Weibull distributions. An example derivation of factors in Table A1.1 is provided in Appendix A1. Note that these equations only position the resistance distribution in terms of $R_{.05}$ – not in relation to the load distribution. The relative positioning of the distributions is accomplished via the actual design checking equation.

Computing R_n normalized to the nominal dead load (D_n)

The previous sections have described how the load and resistance variables must be defined for use in NBS SP577 style reliability analysis. The final step required before performing the analysis is the positioning of the loads and resistances relative to each other via the actual design condition. For the dead plus live load combination, the design checking equation is:

$$\lambda\phi R_n \geq 1.2D_n + 1.6L_n$$

where:

λ = time effect factor

ϕ = resistance factor

R_n = nominal resistance for LRFD (i.e. LRFD reference resistance value as published in the applicable design specification such as the *National Design Specficiation® (NDS®) for Wood Construction*

D_n = nominal dead load effect

L_n = nominal live load effect

Solve for R_n/D_n:

$$\lambda\phi R_n \geq D_n \bullet (1.2 + \frac{1.6L_n}{D_n})$$

Table 3. R_n / D_n across range of Load Ratios (L/D) [1]

L/D =	1	2	3	4	5	6	7	8	9
R_n / D_n	3.294	5.176	7.059	8.941	10.824	12.706	14.588	16.471	18.353

[1] Valid for D+L or D+S load combinations. See equations to compute values for other cases.

$$R_n/D_n \geq (1.2 + \frac{1.6L_n}{D_n})/(\lambda\phi)$$

Substituting $\lambda = 1.0$ (time-effect is not part of this analysis) and $\phi = 0.85$ (for bending) yields the values in Table 3.

Summary

This paper provides the background behind much of the arithmetic required to generate consistent input values for reliability analysis. It describes the basis upon which the random variables of interest (loads and resistance) are derived in a consistent manner and related in the analysis via the design checking equation.

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Appendix A1: Step-by-step calculation of Table A1.1 factors (bending member example)

For flexural members, the allowable design stress is typically derived based on the distribution 5th percentile estimate divided by a reduction factor of 2.1. In equation form:

$$R_{ASD} = \frac{R_{.05}}{2.1}$$

For LRFD, the design value is derived by format conversion per ASTM D5457 as:

$$R_{LRFD} = \frac{2.16 \bullet R_{ASD}}{\phi}$$

Therefore:

$$R_{LRFD} = \frac{2.16 \bullet R_{.05}}{2.1 \bullet \phi}$$

Since $R_{LRFD} = R_n$:

$$R_{LRFD} = \frac{2.16 \bullet R_{.05}}{2.1 \bullet \phi}$$

For a bending member, $\phi = 0.85$, and $R_n = 2.54 \times R_{ASD}$ or $1.21 \times R_{.05}$.

The calculation can proceed by rearranging:

$$\frac{R_{.05}}{R_n} = \frac{(2.1 \bullet \phi)}{2.16}$$

Evaluating the ratio of mean-to-nominal can proceed as follows:

$$\frac{R_M}{R_n} = \frac{R_M}{R_{.05}} \bullet \frac{R_{.05}}{R_n}$$

The ratio of 5th percentile over nominal was provided earlier. The ratio of mean over 5th percentile differs depending on the assumed distribution form - provided in Table A1.1 for normal, lognormal, and 2-parameter Weibull across a range of COV's (for ϕ = 0.85).

An example of the calculation for a normal distribution is:

$$R_{.05} = R_M \bullet (1 - 1.645 \bullet V_R)$$

Or:

$$R_M / R_{.05} = 1/(1 - 1.645 \bullet V_R)$$

Combining with earlier calculations, the mean-to-nominal ratio is:

$$\frac{R_M}{R_n} = \frac{1}{1 - 1.645 \bullet V_R} \bullet \frac{2.1 \bullet \phi}{2.16}$$

Table A1.1 $R_M/R_{.05}$ and R_M/R_n
(For ϕ = 0.85; Assumes that $R_{.05}$ = 1.0)

COV	$R_M/R_{.05}$			R_M/R_n [1]		
	Lognormal	Normal	2P Weibull	Lognormal	Normal	2P Weibull
0.10	1.184	1.197	1.224	0.978	0.989	1.012
0.11	1.205	1.221	1.251	0.996	1.009	1.034
0.12	1.226	1.246	1.279	1.013	1.030	1.057
0.13	1.248	1.272	1.309	1.031	1.051	1.082
0.14	1.270	1.299	1.339	1.050	1.073	1.107
0.15	1.292	1.328	1.370	1.068	1.097	1.132
0.16	1.315	1.357	1.403	1.087	1.121	1.159
0.17	1.339	1.388	1.437	1.107	1.147	1.188
0.18	1.363	1.421	1.472	1.126	1.174	1.216
0.19	1.388	1.455	1.508	1.147	1.202	1.246
0.20	1.413	1.490	1.546	1.168	1.231	1.278
0.21	1.438	1.528	1.585	1.188	1.263	1.310
0.22	1.464	1.567	1.625	1.210	1.295	1.343
0.23	1.491	1.609	1.667	1.232	1.330	1.378
0.24	1.518	1.652	1.711	1.254	1.365	1.414
0.25	1.545	1.699	1.756	1.277	1.404	1.451
0.26	1.574	1.747	1.803	1.301	1.444	1.490
0.27	1.602	1.799	1.851	1.324	1.487	1.530
0.28	1.632	1.854	1.902	1.349	1.532	1.572
0.29	1.662	1.912	1.954	1.373	1.580	1.615
0.30	1.692	1.974	2.008	1.398	1.631	1.659

[1] **R_M/R_n = ($R_M/R_{.05}$) * ($R_{.05}/R_n$) , where $R_{.05}/R_n$ = 2.1ϕ/2.16 ≈ 0.826 (for bending)**

Table A2.1. R_M / D_n for Lognormal Resistance Distribution [1]

COV	Load Ratio (L/D)								
	1	2	3	4	5	6	7	8	9
0.10	3.22	5.06	6.91	8.75	10.59	12.43	14.27	16.12	17.96
0.11	3.28	5.15	7.03	8.90	10.78	12.65	14.53	16.40	18.28
0.12	3.34	5.24	7.15	9.06	10.97	12.87	14.78	16.69	18.59
0.13	3.40	5.34	7.28	9.22	11.16	13.10	15.05	16.99	18.93
0.14	3.46	5.43	7.41	9.38	11.36	13.34	15.31	17.29	19.26
0.15	3.52	5.53	7.54	9.55	11.56	13.57	15.58	17.59	19.60
0.16	3.58	5.62	7.67	9.72	11.76	13.81	15.85	17.90	19.94
0.17	3.64	5.73	7.81	9.89	11.98	14.06	16.14	18.23	20.31
0.18	3.71	5.83	7.95	10.07	12.19	14.31	16.43	18.55	20.67
0.19	3.78	5.94	8.10	10.26	12.42	14.57	16.73	18.89	21.05
0.20	3.85	6.04	8.24	10.44	12.64	14.84	17.03	19.23	21.43
0.21	3.91	6.15	8.39	10.63	12.86	15.10	17.34	19.57	21.81
0.22	3.99	6.26	8.54	10.82	13.10	15.37	17.65	19.93	22.20
0.23	4.06	6.38	8.70	11.02	13.34	15.66	17.97	20.29	22.61
0.24	4.13	6.49	8.86	11.22	13.58	15.94	18.30	20.66	23.02
0.25	4.21	6.61	9.01	11.42	13.82	16.22	18.63	21.03	23.43
0.26	4.28	6.73	9.18	11.63	14.08	16.53	18.98	21.42	23.87
0.27	4.36	6.85	9.35	11.84	14.33	16.82	19.31	21.81	24.30
0.28	4.44	6.98	9.52	12.06	14.60	17.14	19.67	22.21	24.75
0.29	4.52	7.11	9.70	12.28	14.87	17.45	20.04	22.62	25.21
0.30	4.61	7.24	9.87	12.50	15.13	17.77	20.40	23.03	25.66

[1] **R_M/D_n = (R_n/D_n) from Table 3 multiplied by (R_M/R_n) from Table A1.1.**

Where:

V_R = Coefficient of variation for resistance

ϕ = resistance factor

2.1 = reduction factor for ASD

R_M = Mean resistance

$2.16/\phi$ = format conversion factor for LRFD

R_n = nominal resistance

Example for V_R = 0.20:

$$\frac{R_M}{R_n} = \frac{1}{1 - 1.645 \bullet 0.20} \bullet \frac{2.1 \bullet 0.85}{2.16} = 1.231$$

To summarize, $R_M/R_{.05}$ is a function of COV for the assumed distribution type and $R_n/R_{.05}$ is a function of the resistance factor, ϕ. The values for R_M/R_n in Table A1.1 are calculated for ϕ = 0.85 and for $R_{.05}/R_{ASD}$ = 2.1. To adjust to other ϕ's, the R_M/R_n value in the table can be multiplied by $\phi/0.85$, where ϕ is the appropriate value from Table 2 for the property of interest.

Example for V_R = 0.20 and 2P Weibull:

R_M/R_n = 1.278 (as shown in Table A1.1)

For ϕ = 0.80, R_M/R_n = 1.278 x (0.80/0.85) = 1.203

Appendix A2: Actual Input Values for Various Software Methodologies

For NBS SP577-style software, the distribution information is input separate from the nominal value inputs:

1) First, input the distribution information
- Load distribution inputs (D_M/D_n, L_M/L_n, etc.)

Table A2.2. R_M / D_n for Normal Resistance Distribution [1]

COV	Load Ratio (L/D)								
	1	2	3	4	5	6	7	8	9
0.10	3.26	5.12	6.98	8.84	10.71	12.57	14.43	16.29	18.15
0.11	3.32	5.22	7.12	9.02	10.92	12.82	14.72	16.62	18.52
0.12	3.39	5.33	7.27	9.21	11.15	13.08	15.02	16.96	18.90
0.13	3.46	5.44	7.42	9.40	11.38	13.36	15.33	17.31	19.29
0.14	3.54	5.56	7.58	9.60	11.62	13.64	15.66	17.68	19.70
0.15	3.61	5.68	7.75	9.81	11.88	13.94	16.01	18.08	20.14
0.16	3.69	5.80	7.92	10.03	12.14	14.25	16.36	18.47	20.58
0.17	3.78	5.94	8.10	10.26	12.42	14.57	16.73	18.89	21.05
0.18	3.87	6.08	8.29	10.50	12.71	14.92	17.13	19.34	21.55
0.19	3.96	6.22	8.49	10.75	13.01	15.28	17.54	19.80	22.07
0.20	4.06	6.37	8.69	11.01	13.33	15.65	17.96	20.28	22.60
0.21	4.16	6.54	8.91	11.29	13.67	16.04	18.42	20.80	23.17
0.22	4.27	6.70	9.14	11.58	14.02	16.45	18.89	21.33	23.77
0.23	4.38	6.88	9.39	11.89	14.39	16.89	19.40	21.90	24.40
0.24	4.50	7.07	9.64	12.21	14.78	17.35	19.92	22.49	25.06
0.25	4.62	7.27	9.91	12.55	15.20	17.84	20.48	23.13	25.77
0.26	4.76	7.47	10.19	12.91	15.63	18.34	21.06	23.78	26.50
0.27	4.90	7.70	10.49	13.29	16.09	18.89	21.69	24.49	27.28
0.28	5.05	7.93	10.82	13.70	16.58	19.47	22.35	25.24	28.12
0.29	5.20	8.18	11.15	14.13	17.10	20.08	23.05	26.03	29.00
0.30	5.37	8.44	11.52	14.59	17.66	20.73	23.80	26.87	29.94

[1] **R_M/D_n = (R_n/D_n) from Table 3 multiplied by (R_M/R_n) from Table A1.1.**

are precisely per Table 1

- Resistance distribution inputs (R_M/R_n) are input based on actual test data or computed like the values in Table A1.1 (substituting the correct ϕ).

2) Next, input the nominal values

- Nominal value inputs are normalized to the nominal dead load ($D_n = 1.0$):
- For dead load: input 1.0
- For live, snow, or wind load: input the L/D, S/D, or W/D ratio
- For resistance, input the R_n/D_n value in Table 3 corresponding to the load ratio

For other software routines that do not separate the inputs into two steps, everything is the same except for the mean value inputs. To review, the NBS SP577-style software requires input of the mean-to-nominal ratio (i.e., R_M/R_n) separate from the normalized nominal value (i.e., R_M/D_n). In the alternative type of software routine, the mean values are input directly (normalized to D_n). For this reason, the correct mean value resistance inputs for this type of software are the product of Table A1.1 multiplied by Table 3 - provided in Tables A2.1 through A2.3 for three distribution types (for $\phi = 0.85$). Similarly, the correct mean value load inputs (other than dead load) are the mean values from Table 1 multiplied by the load ratio.

Example of NBS SP577 Input Values for Design Case:

- Dead plus Snow (non-regional); Load ratio = 3
- Bending member, Resistance data = 2P Weibull, COV=0.20

Table A2.3. R_M / D_n for 2-Parameter Weibull Resistance Distribution [1]

COV	Load Ratio (L/D)								
	1	2	3	4	5	6	7	8	9
0.10	3.33	5.24	7.14	9.04	10.95	12.85	14.76	16.66	18.56
0.11	3.41	5.35	7.30	9.24	11.19	13.14	15.08	17.03	18.97
0.12	3.48	5.47	7.46	9.45	11.44	13.43	15.42	17.41	19.40
0.13	3.56	5.60	7.64	9.67	11.71	13.74	15.78	17.82	19.85
0.14	3.64	5.73	7.81	9.89	11.98	14.06	16.14	18.23	20.31
0.15	3.73	5.86	7.99	10.12	12.25	14.39	16.52	18.65	20.78
0.16	3.82	6.00	8.18	10.37	12.55	14.73	16.91	19.10	21.28
0.17	3.91	6.15	8.38	10.62	12.85	15.09	17.32	19.56	21.79
0.18	4.01	6.30	8.59	10.88	13.17	15.46	17.75	20.04	22.33
0.19	4.10	6.45	8.80	11.14	13.49	15.83	18.18	20.53	22.87
0.20	4.21	6.61	9.02	11.42	13.83	16.23	18.64	21.04	23.45
0.21	4.31	6.78	9.25	11.71	14.18	16.64	19.11	21.57	24.04
0.22	4.42	6.95	9.48	12.01	14.54	17.06	19.59	22.12	24.65
0.23	4.54	7.13	9.72	12.32	14.91	17.50	20.10	22.69	25.28
0.24	4.66	7.32	9.98	12.64	15.30	17.97	20.63	23.29	25.95
0.25	4.78	7.51	10.24	12.97	15.71	18.44	21.17	23.90	26.63
0.26	4.91	7.71	10.52	13.32	16.13	18.93	21.74	24.54	27.35
0.27	5.04	7.92	10.80	13.68	16.56	19.44	22.31	25.19	28.07
0.28	5.18	8.14	11.10	14.05	17.01	19.97	22.93	25.89	28.85
0.29	5.32	8.36	11.40	14.44	17.48	20.52	23.56	26.60	29.64
0.30	5.47	8.59	11.71	14.84	17.96	21.08	24.21	27.33	30.45

[1] **R_M/D_n = (R_n/D_n) from Table 3 multiplied by (R_M/R_n) from Table A1.1.**

1) Distribution information:

From Table 1:

- $D_M/D_n = 1.05$; COV = 0.10; Normal distribution
- $S_M/S_n = 0.82$; COV = 0.26; Extreme value Type 2 distribution

From Table A1.1:

- $R_M/R_n = 1.278$ (for COV = 0.20 and 2P Weibull distribution

2) Nominal values:

- $D_n = 1.0$; $S_n = 3.0$ (for load ratio = 3)
- $R_n/D_n = 7.059$ (from Table 3 for load ratio = 3)

Example of Alternative Software Input Values (COV and distribution forms unchanged):

- $D_M/D_n = 1.05$
- $S_M/D_n = 2.46$ (derived as follows):
  - $S_M/S_n = 0.82$, multiplied by
  - $S_n/D_n = 3$
  - 0.82 x 3 = 2.46
- $R_M/D_n = 9.018$ (from Table A2.3, derived as follows):
  - R_M/R_n (from Table A1.1) = 1.278, multiplied by
  - R_n/D_n (from Table 3) = 7.059
  - 1.278 x 7.059 = 9.02

Historical Approach Making a Comeback: Closed-Form Equations to Determine LRFD Reliability Indices (β) and Resistance Factors (φ)

By David S. Gromala, PE, Philip Line, PE, Joseph F. Murphy, PhD, Thang Dao, PhD

Overview

Although structural reliability analysis techniques have evolved over the past four decades, several closed-form equations are still referenced in today's building codes and reference standards. These simplified approaches have the advantages of ease of use and broad applicability while retaining most of the accuracy provided by more complex analysis methods.

This article consolidates background information on the most commonly referenced closed-form structural reliability calculation equations found in codes and standards in the U.S. and elsewhere.

Early work

Cornell (1969) proposed a framework for structural codes "in which probability is used to enhance realism and improve consistency" for design of structures. He discussed various aspects of probability and uncertainty in design and showed how load effects and member resistance can be related via equations using specified means and coefficients of variation. Hasofer and Lind (1974), which built upon work by Cornell (1969) and Lind (1971), introduced a general definition of the reliability index. For the linear limit state function, $g(R,S) = R\text{-}S < 0$, where R and S are random variables of resistance and load, respectively, the reliability index can be calculated using the following equation:

$$\beta = \frac{R_m - S_m}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

where:

β = Reliability index

R_m = Mean resistance

S_m = Mean load

σ_R = Standard deviation of R

σ_S = Standard deviation of S

Cornell indicates that this approach does not require specific assumptions regarding distribution forms (types). However, he indicates that the extension of the concept of a "reliability index" to an estimate of probability of failure does indeed require one to assume distribution forms for the variables. More detailed information on this topic is provided by Nowak and Collins (2000).

Hasofer and Lind state that "It does not change the result if we write the failure criterion $(R/S)\text{-}1 < 0$ or $(\log R - \log S) < 0$. As long as the basic variables remain R and S, the reliability is unchanged." To eliminate some of the mathematical difficulties associated with the aforementioned equation (such as the possibility of a negative answer), the failure criterion of $\ln(R/S) < 0$ quickly became more popular. After several intermediate calculations (and modifying the notation for load

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effects from "S" to "Q" (to minimize confusion with other structural notations), this resulted in the familiar equation form:

$$\beta = \frac{\ln\left(\frac{R_m}{Q_m}\right)}{\sqrt{V_R^2 + V_Q^2}}$$

Using the linearization initially proposed by Lind, the denominator can be modified to decouple the load effect from the resistance:

$$\sqrt{V_R^2 + V_Q^2} = \alpha\,(V_R + V_Q)$$

Lind optimized the constant, α, to be ¾ (0.75) with less than 6% linearization error over a broad range of variability (range of $1/3 < V_R/V_Q < 3$). Note that this maximum linearization error is proven to be valid over the entire range of interest for this article (COV's from 10% to 30%)

Simplifying and separating resistances and load effects:

$$R_m \bullet e^{-\alpha\beta V_R} = R_Q \bullet e^{\alpha\beta V_Q}$$

If a design equation is proposed as:

$$\phi R_n \geq \gamma Q_n$$

The equations can be combined as:

$$\phi = \frac{R_m}{R_n} \bullet e^{-\alpha\beta V_R}$$

And:

$$\gamma = \frac{Q_m}{Q_n} \bullet e^{\alpha\beta V_Q}$$

where:

ϕ = Resistance factor

R_m = Mean resistance

R_n = Nominal resistance

α = Constant in square root approximation

V_R = Coefficient of variation of resistance

γ = Load factor

Q_m = Mean load effect

Q_n = Nominal load effect

V_Q = Coefficient of variation of load

Galambos and Ravindra (1973) sought to refine the linearization constant. They separated the single α constant into separate constants related to variability in resistance, each type of load in a combination, and a "structural analysis" factor. After examining a reasonable variability range for these parameters, they generated optimal α's for each. Their results converged to a fairly narrow range of α's, and they proposed α = 0.55 for both load and resistance.

Although this discussion has been provided in a historical context, these equation forms are still in use today. They form the basis of target reliability indices in several countries. Today's ASCE 7-10 (ASCE, 2010) Commentary Section C2.3-2 includes these equation forms – with further refinement of the α's – recommending the use of α = 0.80 for the loads and α = 0.70 for the resistances.

Calculation Steps for Load and Resistance Input Values

There are three sets of calculations required to convert these equations into a form that provides insights for factors affecting calculated reliability index:

1) Compute Q_M as a function of the nominal dead load (D_n), and V_Q as a function of the load ratio (L/D)
2) Compute R_M as a function of R_n, and R_n as a function of D_n
3) Compute the ratio of R_M/Q_M based on the design checking equation.

Step 1. Computing Q_M and V_Q for the dead plus live load combination

Distribution parameters for dead load and live load have been very stable over the years:

$D_M = 1.05D_n$ and $V_D = 0.10$

$L_M = 1.0L_n$ and $V_L = 0.25$

where:

D_M = Mean dead load

V_D = Coefficient of variation of dead load

L_M = Mean live load

V_L = Coefficient of variation of live load

Note: The statistics for other load cases have been updated as more data have been accumulated. In particular, the statistics for snow loads and wind loads have evolved to include region-specific values. The main focus of this article is on dead and live loads. However, the calculations can be extended to other load combinations if desired.

The equations for computing Q_M and V_Q for each load combination follow the concepts outlined by Lin, Yu, and Galambos (1990). The equations differ slightly because that reference computes load ratios as D/L, whereas ASTM D5457 *Standard Specification for Computing Reference Resistance of Wood-Based Materials and Structural Connections for Load and Resistance Factor Design* (ASTM 2015) typically uses the inverse (L/D).

Knowing that the expected value of the sum of independent random variables is equal to the sum of the expected values of the variables (i.e., $E(\Sigma X_i) = \Sigma[E(X_i)]$:

$Q_M = D_M + L_M$

$Q_M = 1.05D_n + L_n$

Similarly, the variance of the sum of independent random variables is equal to the sum of the variances of the variables (i.e., $Var(\Sigma X_i) = \Sigma[Var(X_i)]$):

$$V_Q = \frac{\sqrt{D_M^2 V_D^2 + L_M^2 V_L^2}}{D_M + L_M}$$

Substituting the term L/D:

$$V_Q = \frac{\sqrt{(1.05V_D)^2 + \left(\frac{L_n}{D_n} \bullet V_L\right)^2}}{1.05 + L_n/D_n}$$

The values for Q_M and V_Q for L/D ratios from 1 to 9 are provided in Table 1.

Step 2. Computing R_M based on design value derivations

For many structural materials, determining the ratio of mean strength (R_M) to nominal strength (R_n) is relatively simple, since these values are tabulated in reference documents. However, because published design values for wood LRFD are based on format conversion from ASD, this calculation requires several more steps for wood products.

The calculation starts by relating the LRFD value (the "nominal" value) to the population 5th percentile, followed by relating the mean value to the nominal. The first step is computed by format conversion per ASTM D5457 (2015) and the second step is computed by evaluating the statistics for the assumed distribution form.

Table 1. Q_M / D_n and V_Q across range of Load Ratios (L/D) [1]

L/D =	1	2	3	4	5	6	7	8	9
Q_M / D_n	2.05	3.05	4.05	5.05	6.05	7.05	8.05	9.05	10.05
V_Q	0.132	0.168	0.187	0.150	0.207	0.213	0.218	0.221	0.224

[1] Valid for D+L or D+S load combinations. See equations to compute values for other cases.

For bending, the allowable design stress is typically derived based on the distribution 5th percentile estimate divided by a reduction factor of 2.1. In equation form:

$$R_{ASD} = \frac{R_{.05}}{2.1}$$

The LRFD reference resistance, R_{LRFD}, is derived by format conversion per ASTM D5457 as:

$$R_{LRFD} = \frac{2.16 \bullet R_{ASD}}{\phi}$$

Therefore:

$$R_{LRFD} = \frac{2.16 \bullet R_{.05}}{2.1 \bullet \phi}$$

Since $R_{LRFD} = R_n$:

$$R_n = \frac{2.16 \bullet R_{.05}}{2.1 \bullet \phi}$$

Rearranging:

$$\frac{R_{.05}}{R_n} = \frac{(2.1 \bullet \phi)}{2.16}$$

For the second step (evaluating the ratio of mean-to-nominal), the calculation can be separated into two factors:

$$\frac{R_M}{R_n} = \frac{R_M}{R_{.05}} \bullet \frac{R_{.05}}{R_n}$$

The ratio of 5th percentile over nominal differs for each resistance factor (ϕ) as shown previously. The ratio of mean over 5th percentile differs depending on COV and the assumed distribution form.

A simple example, using a normal distribution:

$$R_{.05} = R_M \bullet (1 - 1.645 \bullet V_R)$$

Or:

$$R_M / R_{.05} = 1/(1 - 1.645 \bullet V_R)$$

Thus, for a normal distribution:

$$\frac{R_M}{R_n} = \frac{1}{1 - 1.645 \bullet V_R} \bullet \frac{2.1 \bullet \phi}{2.16}$$

The ratios $R_M/R_{.05}$ and R_M/R_n are computed in a similar manner for other distributions. Note that these equations only position the resistance distribution in terms of $R_{.05}$ – not in relation to the load distribution. The relative positioning of the distributions is accomplished via the actual design checking equation and is provided in Step 3.

Step 3 is only required if one wishes to compute the reliability index (β) for a specific resistance case using input values for R_M/R_n and COV. One could also choose to solve the closed-form equation or the ASCE 7-10 *Minimum Design Loads for Buildings and Other Structures* (ASCE, 2010) equation for a specific target β – solving for the minimum required R_M/R_n that will achieve that β across a range of COV's.

Step 3. Position the distributions (i.e., compute R_M/Q_M)

The load and resistance distributions are positioned relative to each other via the actual design condition. For the dead plus live load combination, the design checking equation is:

$$\lambda \phi R_n \geq 1.2 D_n + 1.6 L_n$$

Solve for R_n/D_n:

Table 2. R_n / D_n across range of Load Ratios (L/D) [1]

L/D =	1	2	3	4	5	6	7	8	9
R_n / D_n	3.294	5.176	7.059	8.941	10.824	12.706	14.588	16.471	18.353

[1] Valid for D+L or D+S load combinations. See equations to compute values for other cases.

$$\lambda \phi R_n \geq D_n \bullet (1.2 + \frac{1.6L_n}{D_n})$$

$$R_n/D_n \geq (1.2 + \frac{1.6L_n}{D_n})/(\lambda\phi)$$

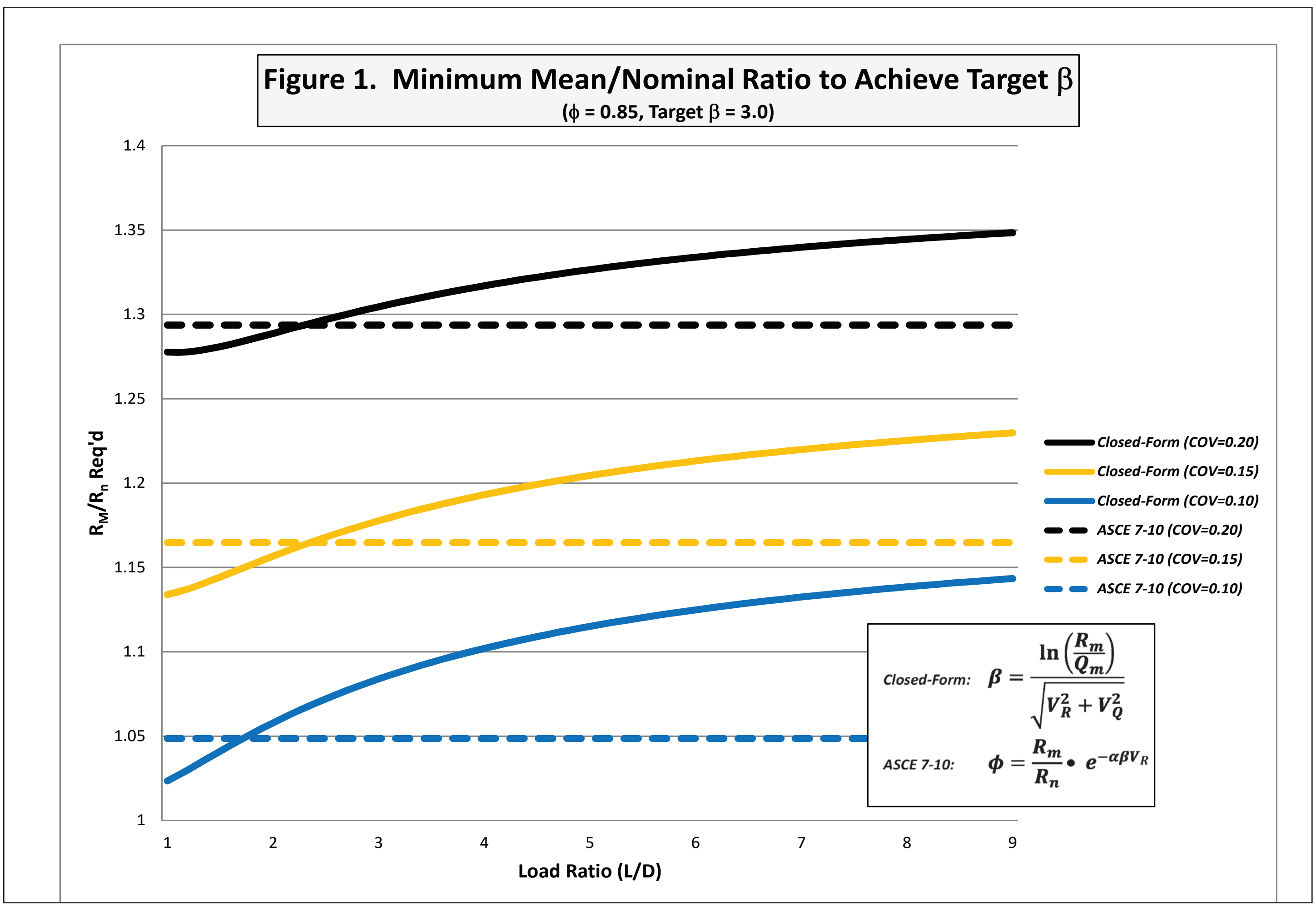
Substituting λ = 1.0 (time-effect is not part of this analysis) and ϕ = 0.85 (for bending) yields the values in Table 2.

Combining the results of the three calculation steps provided above yields R_M/Q_M:

$$\frac{R_M}{Q_M} = \frac{R_M}{R_n} \bullet \frac{R_n}{D_n} \div \frac{Q_M}{D_n}$$

Presentation of Results

The closed-form equations can be useful in various ways. For example, one can compute the minimum required R_M/R_n that will provide a specified target β across a range of COV's. Figure 1 illustrates that there are differences between the standard closed-form



equation and the ASCE 7-10 version. Note that these differences are to be expected, since this specific ASCE 7-10 equation only deals with the resistance. Readers should note that the ASCE 7-10 equation is intended to be used in context with ASCE 7-10 load factors and load combinations and specified material strengths in accordance with applicable design standards. As noted in underlying references, the equation is based on several broad simplifying assumptions. The equation itself does not address the level or method of testing, design criteria, and end use conditions addressed by applicable design standards such as the *National Design Specification® (NDS®) for Wood Construction* to produce satisfactory designs.

These results can be extended by examining a range of reference cases that use the same assumptions that formed the basis of the original ASTM D5457 standard. When ASTM D5457 generated its estimated "representative" reliability index (β) of 2.4 for wood products, it was based on several key assumptions (some of which provide conservative estimates):

1) The underlying resistance distribution is 2-parameter Weibull,
2) The 5^{th}-percentile of the distribution is equal to the minimum permitted by the underlying product standard (for example, $R_{.05} = 2.1 \times R_{ASD}$ for bending), and
3) All other assumptions and methods used in NBS SP577 (Ellingwood, et. al, 1980) are valid.

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Summary

The current equation recommended within ASCE 7-10 to establish resistance factors has its roots in the closed-form equations of the late 1960s and early 1970s. This paper traces the various forms of the historical equations to assist the reader to better understand the basis of these equations.

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