

Handout for ASTM D07.02.06 Reliability-Based Design, October 2019

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Ctrl-Shift"-" rotates viewing window counter-clockwise.

Presentation

My thoughts on wood load-time factors	Pg 2-3
Overlay of ASD Duration of Load factors C_D on wood's Curve	Pg 4
Probability density functions, green - dead load pdf, red - live load pdf, blue - resistance pdf with 15% CoV, magenta - resistance pdf with 25% CoV. Both resistance distributions have the same 5th percentile value of 8.4 . 15% CoV has reliability index 3.36, 25% CoV has reliability index 2.84 .	Pg 5
Five Reliability Index Comparisons	Pg 6-8

Not presented but useful for Discussion

First-order second-moment, FOSM, Reliability Index Failure versus Reliability	Pg 9
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A possible future direction of D5457	Pg 11
More thoughts on reliability based design	Pg 12

Useful references

Lyman W. Wood's "Relation of Strength of wood to Duration of Load" December 1951, No. R1916 FPL, FS, USDA This report started it all.	Pg 13-22
Basic statistics for summing expected values and variances	Pg 23-26
Residential Structural Design Guide 2000, Chapter 5	Pg 27-30

28 September 2019

My thoughts on wood load-time factors -- Joe Murphy

- A safety checking equation is a linear combination of (nominal) numbers *separated* using factors.
- Different safety equations or different formats are different linear combinations of nominals and factors.
- A strict conversion from one format to another does not change reliability.
- ASD considers the 5th-percentile of the resistance distribution along with the load nominals in the safety equation.

Products with the same 5th-percentile are considered equal. Products must meet a minimum 5th-percentile value determined by the safety checking equation.

In ASD the 5th-percentile value is the metric to compare products and safety.

- Reliability analysis takes into account *variability* in loads and resistance. Reliability analysis uses load and resistance *distributions*. Reliability analysis can be applied to any safety checking equation. The linear combination of nominals and factors *separate* distributions.

Reliability analysis yields a reliability index β

Different methods of reliability analyses produce different reliability indices.

- Reliability-based LRFD considers the resistance distribution along with the load distributions and the safety checking equation.

Products with the same reliability index β are considered equal. Products must meet a minimum β value determined by reliability analysis.

In reliability-based LRFD the β value is the metric to compare products and safety.

- Adjustment factors for design values.

Typically in research, one takes two samples of wood product (doing everything possible to make sure the two samples are identical in sample size and strength distribution). Then one sample is the "control" sample and the other is treated in the generic sense and is the "treated" sample. Both control and treated samples are tested and the means of the tests are calculated and compared using e.g., Student's t-test.

If the difference is considered significant the ratio of treated mean to control mean is considered an adjustment factor and is applied to every percentile of the distribution. The mean-to-5th-percentile ratio and the coefficient of variation is the same for both a treated distribution and a nontreated distribution. See pages 23-26.

- In reliability analyses all adjustments are considered to have been applied to the resistance distribution, R .
- Load-time factors.

70 years ago Lyman W. Wood conducted research, pages 13-22, where he estimated wood strength of small clear wood specimens and applied a load less than the estimated strength. This produced a curve, Figure 1, with increasing time-to-failure the lower the ratio of stress-to-strength. This stress-to-strength ratio is a load-time factor.

The load-time factor was applied to the 5th-percentile used in ASD, pages 27-30.

If the live load is the predominant load, e.g., live load nominal is 3 times dead load nominal, then, by consensus, the duration of the sum of load nominals was thought not to exceed 10 years over a 50 year lifetime period. To prevent static fatigue, in this load case, the stress ratio, i.e., load-time factor, on the resistance 5th-percentile was made $10/16 = 62\%$ according to wood's curve, page 4, and ASD, page 27. Using the same logic and consensus, durations and load-time factors were assigned to other predominant load cases.

Take Wood's Figure 1 and put the independent variable on the horizontal axis, page 4. Now take a distribution of strength. Applying a load, the 5th-percentile's stress ratio would give an acceptable estimated duration for that load over a 50 year lifetime. Strength values greater than the 5th have stress ratios less than the 5th, and that load would last longer, and perhaps much longer, than the 5th. Consider the mean of the strength distribution which might be 1.5 times the 5th. The stress ratio of the mean would be 67% of the 5th-percentile stress ratio.

Because the load-time factor, as applied to the 5th-percentile, cannot be applied to the mean (or other percentiles), the load-time factor cannot be considered an adjustment factor.

- I consider the load-time factor to be a *separation* factor. As the load-time factor decreases the resistance 5th-percentile separates farther from the ASD load nominals and the resistance distribution separates farther from the load distributions in reliability-based LRFD. See page 5.
- A change in the load-time factor changes the reliability and reliability index. A decrease in the load-time factor decreases the stress ratio and increases the reliability and the reliability index.
- Excluding the load-time factor from the safety checking equation means that the calculated reliability index is the same no matter the numerical value of the load-time factor. And the resistance distribution does not separate farther from the load distributions with a decrease in load-time factor.
- with an appropriate low load-time factor, an exceedingly small percentage will have a high enough stress ratio to fail by static fatigue (compared to overload), which is witnessed by historic observation.

On pages 6-8 the reliability index is calculated 5 different ways. The reliability indices are calculated in tables for the: 1) LRFD safety checking equation, K_R , reliability-based; 2) LRFD safety checking equation, K_F , format conversion, and; 3) ASD safety checking equation.

Page 7 has the three tables with the load-time factors considered as *separation* factors.

Page 8 has the three tables with the load-time factors considered as *adjustment* factors (i.e., not in the safety checking equation.)

C_D

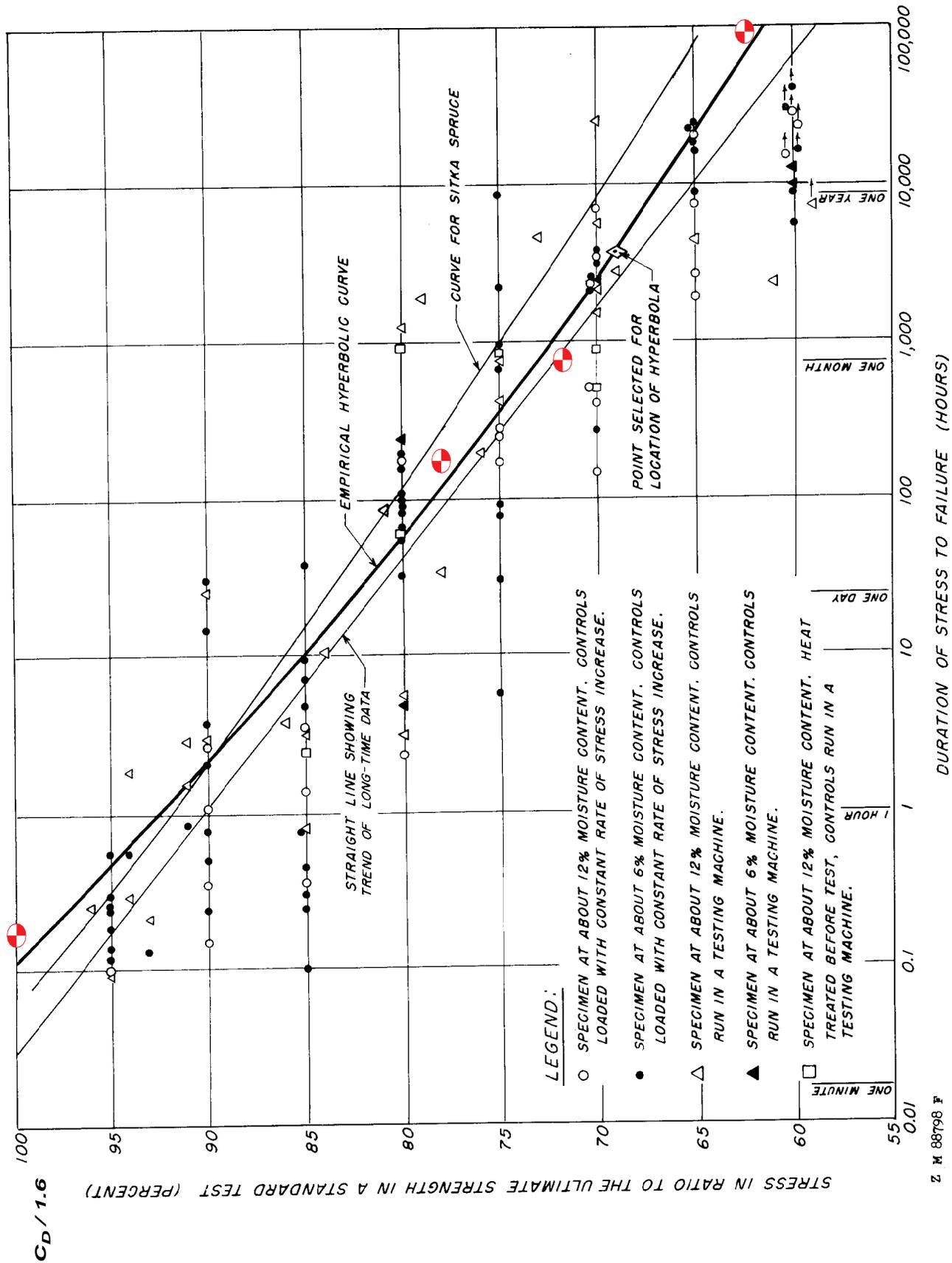
1.6 10 minutes

1.25 7 days

1.15 1 month

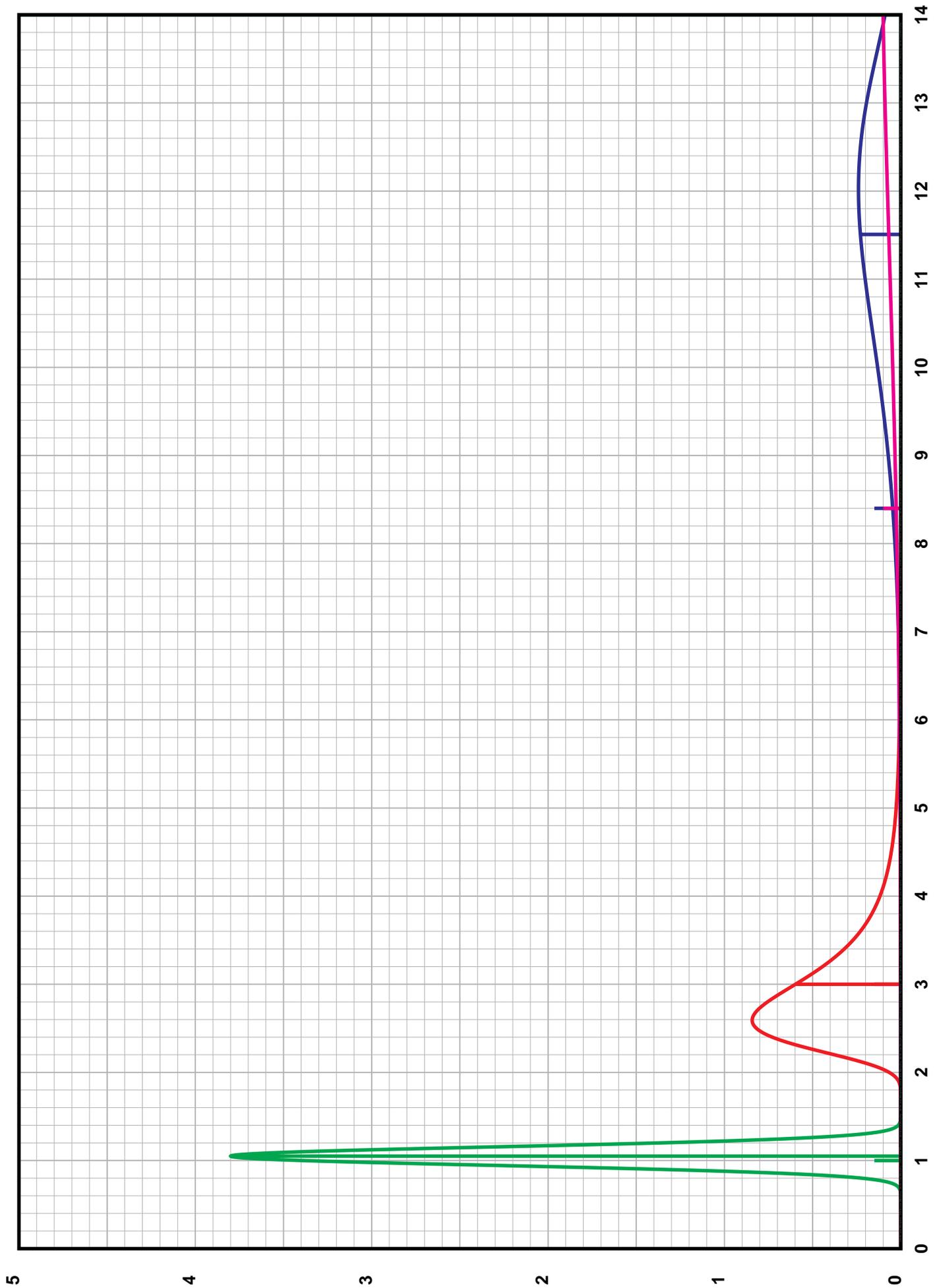
1. 10 years

0.9 50 Yea



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Figure 1.--Relation of duration of constant stress to level of stress in long-time loading of Douglas-fir bending specimens.



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Five Reliability Index Comparisons by Joe Murphy

The R_{05} I use is the fifth percentile of the initial strength distribution. I also use the following statistics. The load statistics come from NBS Special Publication 577. The coefficient of variations, CVs, for resistance come from the 2010 Wood Handbook Table 5-6 for clear wood.

Distribution Name	Load Distribution	$\frac{\bar{X}}{X_n}$	CV_X	Distribution Name	Resistance Distribution	$\frac{\bar{R}}{R_{05}}$	CV_R
Dead	normal	1.05	0.10	R_Compression	weibull III	1.472	0.18
Live	Gumbel I	1.00	0.25	R_Bending	weibull III	1.403	0.16
Snow	Frechet II	0.82	0.26	R_Tension	weibull III	1.756	0.25
wind	Gumbel I	0.78	0.37	R_Shear	weibull III	1.339	0.14

I calculate the following expected values and variances, Vs:

$$\bar{R} = \left(\frac{\bar{R}}{R_{05}} \right) R_{05} \text{ and } V_R = (\bar{R} CV_R)^2$$

$$\bar{D} = \left(\frac{\bar{D}}{D_n} \right) D_n \text{ and } V_D = (\bar{D} CV_D)^2 \text{ and } \bar{X} = \left(\frac{\bar{X}}{X_n} \right) X_n \text{ and } V_X = (\bar{X} CV_X)^2$$

and define

$$\bar{Q} = \bar{D} + \bar{X} \text{ and } V_Q = V_D + V_X \text{ and } CV_Q = \frac{\sqrt{V_Q}}{\bar{Q}} \text{ and } CV_Q^2 = \frac{V_D + V_X}{(\bar{D} + \bar{X})^2}$$

Each cell on the next two pages contain 5 β s, and R_{05} when $D_n = 1$. The 5 β s are:

β_1	β_2
β_3	R_{05}
β_4	β_5

The β s are calculated:

$$\beta_1 = \frac{\bar{R} - \bar{Q}}{\sqrt{V_R + V_Q}} \text{ and } \beta_2 = \frac{\ln(\bar{R} / \bar{Q})}{\sqrt{CV_R^2 + CV_Q^2}} \text{ where } R \text{ and } Q \text{ are independent random variables. Distribution types not needed.}$$

β_3 , uses the Advanced First-Order Second-Moment, AFOSM, reliability procedure used in NBS SP 577 and NBS577js.html specified distribution types.

β_4 and β_5 were derived using 100,000 Monte Carlo simulations of the random variables (with specified distribution types) to produce a g distribution of $g = R - Q = R - (D + X)$

$$\beta_4 = \Phi^{-1} \left(1 - \frac{n(g \leq 0)}{100,000} \right) \text{ where } \Phi^{-1}() \text{ is the inverse CDF of the standardized normal distribution.}$$

$$\beta_5 = \frac{\bar{g}}{\sqrt{V_g}} \quad \beta_5 \text{ and } \beta_1 \text{ should calculate to the same value.}$$

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ASTM D5457 test based Reliability Index (FOSM/AFOSM/Monte Carlo)						
β 2-P weibull ↓ Loads, λ ↓	Compression $CV_R = 0.18$ $\phi_s = 0.90$ $K_R = 1.24$	Bending 0.16 0.85 1.23	Tension 0.25 0.80 1.15	Shear 0.14 0.75 1.40	Nominal safety checking equation include load-time factors	
1.4 D, 0.6	3.60 5.22 3.19 2.091 3.19 3.60	4.07 5.79 3.48 2.232 3.48 4.05	3.04 5.37 3.08 2.536 3.07 3.03	4.48 6.05 3.67 2.222 3.60 4.49	$\phi_s [\lambda K_R R_{05}] \geq 1.4 D_n$	
1.2 D + 1.6 L, 0.8	3.02 3.44 2.74 6.720 2.68 3.01	3.38 3.70 2.96 7.174 2.88 3.36	2.81 4.04 2.80 8.152 2.74 2.80	3.59 3.68 3.05 7.143 2.93 3.58	$\phi_s [\lambda K_R R_{05}] \geq 1.2 D_n + 1.6 L_n, \frac{L_n}{D_n} = 3$	
1.2 D + 1.6 S, 0.8	3.37 4.02 3.07 6.720 2.87 3.36	3.78 4.31 3.26 7.174 3.09 3.80	2.97 4.52 3.04 8.152 2.93 2.97	4.07 4.33 3.25 7.143 3.09 4.08	$\phi_s [\lambda K_R R_{05}] \geq 1.2 D_n + 1.6 S_n, \frac{S_n}{D_n} = 3$	
1.2 D + 1.0 W, 1.0	1.68 1.67 1.67 2.867 1.59 1.68	1.86 1.80 1.82 3.061 1.71 1.86	2.14 2.53 2.10 3.478 2.04 2.14	1.80 1.69 1.76 3.048 1.64 1.79	$\phi_s [\lambda K_R R_{05}] \geq 1.2 D_n + 1.0 W_n, \frac{W_n}{D_n} = 2$	

ASTM D5457 format conversion Reliability Index (FOSM/AFOSM/Monte Carlo)						
β 2-P weibull ↓ Loads, λ ↓	Compression $CV_R = 0.18$ $\phi_s = 0.90$ $K_F = 2.40$ ASTM = 1.9	Bending 0.16 0.85 2.54 2.1	Tension 0.25 0.80 2.70 2.1	Shear 0.14 0.75 2.88 2.1	Nominal safety checking equation include load-time factors	
1.4 D, 0.6	3.56 5.13 3.16 2.052 3.19 3.58	4.10 5.88 3.51 2.270 3.57 4.09	2.93 4.95 2.93 2.269 3.00 2.94	4.54 6.18 3.72 2.269 3.75 4.53	$\phi_s \left[\lambda K_F \frac{R_{05}}{ASTM} \right] \geq 1.4 D_n$	
1.2 D + 1.6 L, 0.8	2.97 3.37 2.70 6.597 2.64 2.98	3.43 3.77 3.00 7.295 2.92 3.44	2.66 3.69 2.63 7.292 2.61 2.66	3.66 3.77 3.10 7.292 3.04 3.64	$\phi_s \left[\lambda K_F \frac{R_{05}}{ASTM} \right] \geq 1.2 D_n + 1.6 L_n, \frac{L_n}{D_n} = 3$	
1.2 D + 1.6 S, 0.8	3.33 3.95 3.03 6.597 2.87 3.34	3.82 4.38 3.29 7.295 3.08 3.82	2.85 4.16 2.89 7.292 2.79 2.86	4.13 4.41 3.29 7.292 3.11 4.15	$\phi_s \left[\lambda K_F \frac{R_{05}}{ASTM} \right] \geq 1.2 D_n + 1.6 S_n, \frac{S_n}{D_n} = 3$	
1.2 D + 1.0 W, 1.0	1.62 1.61 1.62 2.815 1.52 1.61	1.93 1.87 1.86 3.113 1.77 1.93	1.92 2.20 1.90 3.111 1.84 1.92	1.88 1.76 1.82 3.111 1.74 1.89	$\phi_s \left[\lambda K_F \frac{R_{05}}{ASTM} \right] \geq 1.2 D_n + 1.0 W_n, \frac{W_n}{D_n} = 2$	

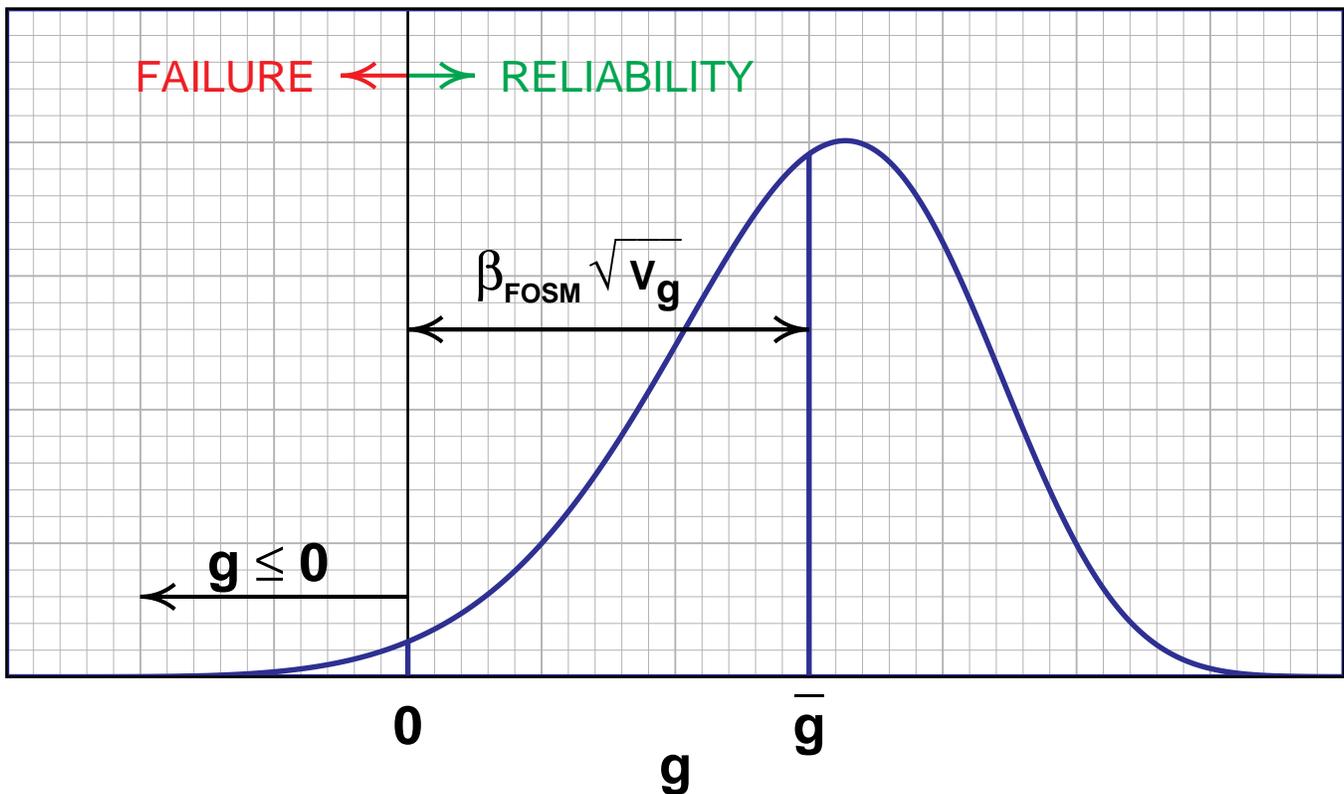
ASD format Reliability Index (FOSM/AFOSM/Monte Carlo)						
β 2-P weibull ↓ Loads, C_D ↓	Compression $CV_R = 0.18$ ASTM = 1.9	Bending 0.16 2.1	Tension 0.25 2.1	Shear 0.14 2.1	Nominal safety checking equation include load-time factors	
D, 0.9	3.62 5.27 3.21 2.111 3.23 3.61	4.16 6.03 3.57 2.333 3.62 4.15	2.96 5.06 2.97 2.333 2.97 2.97	4.61 6.34 3.78 2.333 3.85 4.62	$\left[C_D \frac{R_{05}}{ASTM} \right] \geq D_n$	
D + L, 1.0	3.32 3.91 2.98 7.600 2.95 3.31	3.81 4.34 3.30 8.400 3.26 3.81	2.84 4.14 2.84 8.400 2.83 2.85	4.12 4.37 3.43 8.400 3.34 4.11	$\left[C_D \frac{R_{05}}{ASTM} \right] \geq D_n + L_n, \frac{L_n}{D_n} = 3$	
D + S, 1.15	3.33 3.95 3.04 6.609 2.88 3.33	3.82 4.39 3.29 7.304 3.07 3.81	2.85 4.17 2.89 7.304 2.76 2.86	4.14 4.42 3.30 7.304 3.16 4.15	$\left[C_D \frac{R_{05}}{ASTM} \right] \geq D_n + S_n, \frac{S_n}{D_n} = 3$	
D + 0.6 W, 1.6	1.36 1.35 1.41 2.612 1.32 1.35	1.65 1.59 1.65 2.887 1.55 1.65	1.76 1.98 1.75 2.887 1.68 1.76	1.57 1.48 1.59 2.887 1.48 1.57	$\left[C_D \frac{R_{05}}{ASTM} \right] \geq D_n + 0.6 W_n, \frac{W_n}{D_n} = 2$	

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ASTM D5457 test based Reliability Index (FOSM/AFOSM/Monte Carlo)						
β 2-P weibull ↓ Loads ↓	Compression $CV_R = 0.18$ $\phi_s = 0.90$ $K_R = 1.24$	Bending 0.16 0.85 1.23	Tension 0.25 0.80 1.15	Shear 0.14 0.75 1.40	Nominal safety checking equation Exclude load-time factors	
1.4 D	2.28 2.74 2.08 1.254 2.08 2.29	2.60 3.08 2.31 1.339 2.29 2.59	2.40 3.47 2.32 1.522 2.30 2.39	2.71 3.08 2.37 1.333 2.37 2.72	$\phi_s [K_R R_{05}] \geq 1.4 D_n$	
1.2 D + 1.6 L	2.39 2.58 2.24 5.376 2.15 2.38	2.68 2.79 2.43 5.739 2.33 2.66	2.50 3.33 2.46 6.522 2.41 2.50	2.74 2.72 2.46 5.714 2.36 2.74	$\phi_s [K_R R_{05}] \geq 1.2 D_n + 1.6 L_n, \frac{L_n}{D_n} = 3$	
1.2 D + 1.6 S	2.81 3.15 2.62 5.376 2.44 2.80	3.15 3.40 2.81 5.739 2.62 3.17	2.71 3.81 2.73 6.522 2.61 2.70	3.31 3.36 2.80 5.714 2.62 3.31	$\phi_s [K_R R_{05}] \geq 1.2 D_n + 1.6 S_n, \frac{S_n}{D_n} = 3$	
1.2 D + 1.0 W	1.68 1.67 1.67 2.867 1.59 1.68	1.86 1.80 1.82 3.061 1.71 1.86	2.14 2.53 2.10 3.478 2.04 2.14	1.80 1.69 1.76 3.048 1.64 1.79	$\phi_s [K_R R_{05}] \geq 1.2 D_n + 1.0 W_n, \frac{W_n}{D_n} = 2$	

ASTM D5457 format conversion Reliability Index (FOSM/AFOSM/Monte Carlo)						
β 2-P weibull ↓ Loads ↓	Compression $CV_R = 0.18$ $\phi_s = 0.90$ $K_F = 2.40$ ASTM = 1.9	Bending 0.16 0.85 2.54 2.1	Tension 0.25 0.80 2.70 2.1	Shear 0.14 0.75 2.88 2.1	Nominal safety checking equation Exclude load-time factors	
1.4 D	2.22 2.65 2.03 1.231 2.03 2.24	2.66 3.17 2.35 1.362 2.33 2.66	2.21 3.06 2.12 1.361 2.13 2.22	2.80 3.20 2.43 1.361 2.41 2.79	$\phi_s \left[K_F \frac{R_{05}}{ASTM} \right] \geq 1.4 D_n$	
1.2 D + 1.6 L	2.34 2.51 2.19 5.278 2.12 2.34	2.73 2.86 2.47 5.836 2.38 2.74	2.32 2.97 2.28 5.833 2.24 2.32	2.83 2.81 2.51 5.833 2.43 2.82	$\phi_s \left[K_F \frac{R_{05}}{ASTM} \right] \geq 1.2 D_n + 1.6 L_n, \frac{L_n}{D_n} = 3$	
1.2 D + 1.6 S	2.76 3.08 2.58 5.278 2.41 2.77	3.20 3.47 2.84 5.836 2.66 3.20	2.55 3.45 2.56 5.833 2.47 2.56	3.38 3.45 2.84 5.833 2.67 3.40	$\phi_s \left[K_F \frac{R_{05}}{ASTM} \right] \geq 1.2 D_n + 1.6 S_n, \frac{S_n}{D_n} = 3$	
1.2 D + 1.0 W	1.62 1.61 1.62 2.815 1.52 1.61	1.93 1.87 1.86 3.113 1.77 1.93	1.92 2.20 1.90 3.111 1.84 1.92	1.88 1.76 1.82 3.111 1.74 1.89	$\phi_s \left[K_F \frac{R_{05}}{ASTM} \right] \geq 1.2 D_n + 1.0 W_n, \frac{W_n}{D_n} = 2$	

ASD format Reliability Index (FOSM/AFOSM/Monte Carlo)						
β 2-P weibull ↓ Loads ↓	Compression $CV_R = 0.18$ ASTM = 1.9	Bending 0.16 2.1	Tension 0.25 2.1	Shear 0.14 2.1	Nominal safety checking equation Exclude load-time factors	
D	2.11 2.48 1.93 1.188 1.93 2.10	2.53 2.97 2.25 1.312 2.24 2.53	2.14 2.92 2.05 1.312 2.07 2.15	2.64 2.99 2.32 1.312 2.33 2.64	$\left[1.6 \frac{R_{05}}{ASTM} \right] \geq D_n$	
D + L	2.00 2.10 1.93 4.750 1.84 2.00	2.37 2.43 2.20 5.250 2.10 2.37	2.13 2.63 2.09 5.250 2.06 2.14	2.40 2.36 2.21 5.250 2.10 2.40	$\left[1.6 \frac{R_{05}}{ASTM} \right] \geq D_n + L_n, \frac{L_n}{D_n} = 3$	
D + S	2.46 2.67 2.35 4.750 2.18 2.46	2.87 3.03 2.61 5.250 2.42 2.86	2.38 3.11 2.39 5.250 2.28 2.39	2.99 3.00 2.60 5.250 2.45 3.00	$\left[1.6 \frac{R_{05}}{ASTM} \right] \geq D_n + S_n, \frac{S_n}{D_n} = 3$	
D + 0.6 W	1.36 1.35 1.41 2.612 1.32 1.35	1.65 1.59 1.65 2.887 1.55 1.65	1.76 1.98 1.75 2.887 1.68 1.76	1.57 1.48 1.59 2.887 1.48 1.57	$\left[1.6 \frac{R_{05}}{ASTM} \right] \geq D_n + 0.6 W_n, \frac{W_n}{D_n} = 2$	



we define the random variable g as: $g = R - Q$

where R is the resistance effect random variable and Q is the loads effect random variable (sum of a number of load effect random variables, e.g., $Q = D + X$.)

From statistics we can add the expected values: $\bar{g} = \bar{R} - \bar{Q}$

If the variables are *independent* of each other, we can add the variances: $V_g = V_R + V_Q$

The First-Order Second-Moment reliability index, β_{FOSM} , is:
$$\beta_{\text{FOSM}} = \frac{\bar{g}}{\sqrt{V_g}} = \frac{\bar{R} - \bar{Q}}{\sqrt{V_R + V_Q}}$$

$\beta_{\text{FOSM}} = \beta_1$ when calculated using distribution parameters. $\beta_{\text{FOSM}} = \beta_5$ when calculated using Monte Carlo sampling. β_5 will be slightly less than β_1 because in my Monte Carlo the loads are not allowed to go negative, biasing the loads slightly higher.

Note that the FOSM reliability index (β_1) is valid for any distribution type when calculated using distribution parameters. It (β_1) only uses the means and variances of the distributions!

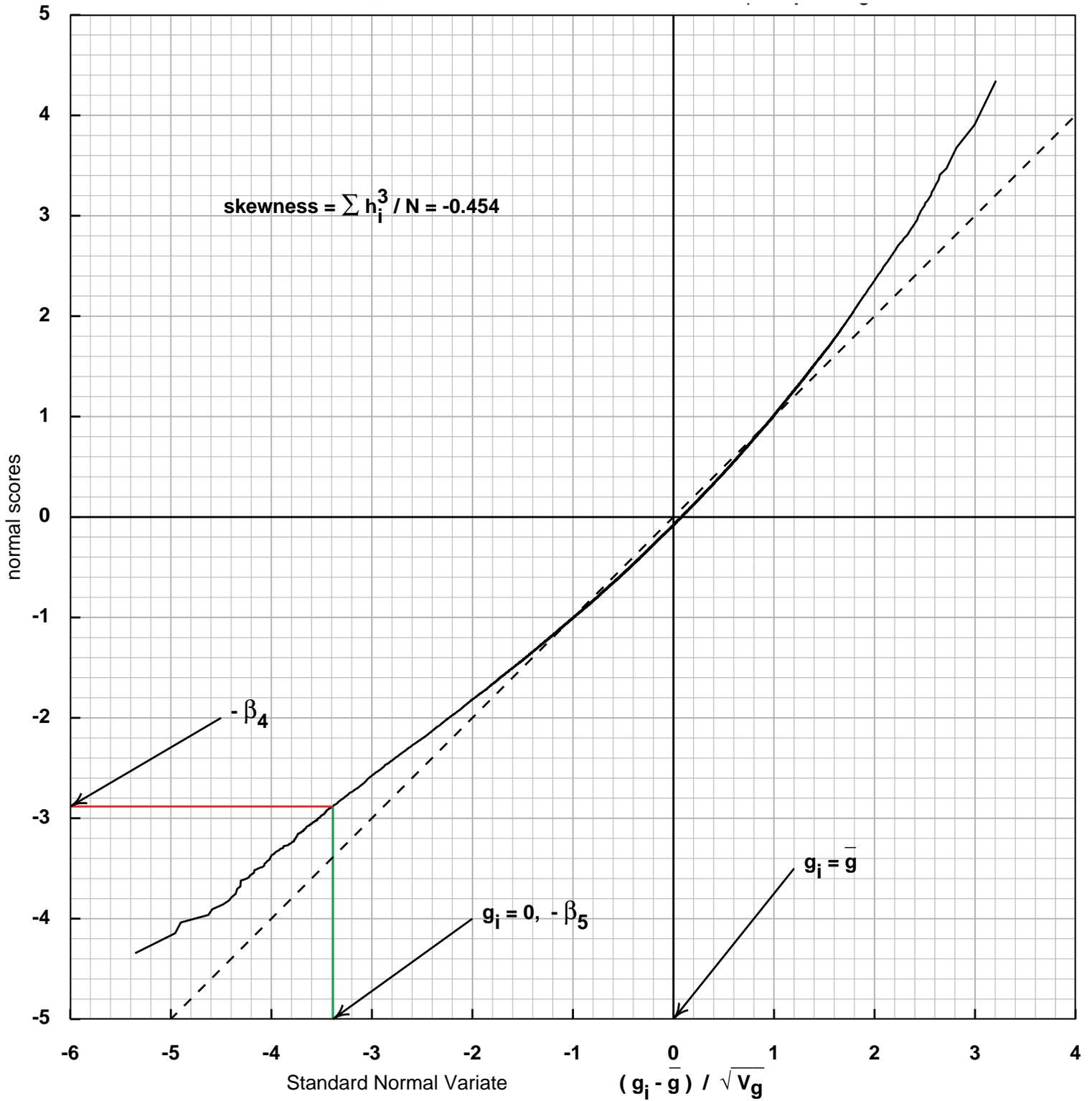
In simulation studies, variable values are chosen at random from the distributions (e.g., Monte Carlo). \bar{g} and V_g are calculated. The probability of $g \leq 0$ is tabulated:

$\text{Prob}[g \leq 0]$ is referred to as probability of failure p_f .

one can then define a reliability index for simulation, $\beta_{\text{Simulated}} (= \beta_4)$, as:

$\beta_{\text{Simulated}} = \Phi^{-1}(1 - p_f)$ where $\Phi^{-1}()$ is the inverse CDF of the standardized normal distribution.

But $\beta_{\text{Simulated}} \neq \beta_{\text{FOSM}}$ except in the very rare case that g is normally distributed.



The limit state equation is $g = R - (D + X)$ In simulation studies, variable values are chosen at random from the distributions R, D, and X (e.g., Monte Carlo) and the sample mean \bar{g} and sample variance V_g are calculated. The probability of failure, p_f is $\text{Prob}[g \leq 0]$ and is the $\text{cdf}_g(0)$:

One can then define a reliability index, β_5 , using the sample mean and variance: $\beta_5 = \bar{g} / \sqrt{V_g}$.

One can also define a reliability index, β_4 , using probability of failure or the cdf of g : $-\beta_4 = \Phi^{-1}(p_f) = \Phi^{-1}(\text{cdf}_g(0))$ where $\Phi^{-1}()$ is the inverse CDF of the standard normal distribution.

If one transforms the g distribution into standard normal variates, $h_i = (g_i - \bar{g}) / \sqrt{V_g}$ one gets the relationship $-\beta_4 = \Phi^{-1}(\text{cdf}_h(-\beta_5))$ This relationship is shown in the above plotted example.

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A possible future direction -- Joe Murphy

ASD and LRFD load-time factors

Harmonize the LRFD load-time factors the same as historically successful ASD, e.g., $\lambda = C_D/1.6$,

ASD and LRFD load-time factors					
Factor	50 yrs	10 yrs	1 mos	7 days	10 min
C_D	0.90	1.00	1.15	1.25	1.60
$\lambda = C_D/1.6$	0.56	0.62	0.72	0.78	1.00

This harmonization leads to simple development of the D5457 format conversion factor K_F and the D5457 reliability normalization factor K_R as follows.

D5457 format conversion factor, K_F

Calculate the format conversion factor, K_F , using the live load case with $L_n/D_n = 3$

$$\frac{\phi_s \lambda K_F R_{05conv}/ASTM}{C_D R_{05asd}/ASTM} = \frac{1.2(1) + 1.6(3)}{1 + 3} = 1.5. \text{ with format conversion } R_{05conv} = R_{05asd} \text{ and } K_F = \frac{2.4}{\phi_s}$$

This yields same size members in both ASD and D5457 format conversion for this case.

D5457 reliability normalization factor, K_R

For D5457 reliability normalization factor, K_R , use the same live load case with $L_n/D_n = 3$. The safety checking equation is

$$\phi_s \lambda K_R R_{05lrfd} = 1.2(1) + 1.6(3) = 6$$

Develop K_R so that there is a constant reliability index $\beta = 3$ for bending with coefficients of variation CV_R from 0.1 to 0.3 (2p weibull resistance distribution) and $\phi_s = 0.85, \lambda = 0.62$

The following table gives the reliability index β_3 for four stress modes. The reliability index was calculated using the advanced first-order second-moment AFOSM methodology used in NBS SP 577 and NBS577js.html

reliability index, β_3						
K_R	1.696	1.592	1.408	1.212	1.029	
$\phi_s \setminus CV_R$	0.10	0.15	0.20	0.25	0.30	
Compression 0.90	2.81	2.86	2.90	2.92	2.93	
Bending 0.85	3.00	3.00	3.00	3.00	3.00	
Tension 0.80	3.20	3.14	3.10	3.08	3.07	
Shear 0.75	3.40	3.28	3.21	3.17	3.14	

Note that in the table the reliability index β goes up as the resistance factor ϕ_s goes down which reflects the $\beta - \phi$ relationship.

28 September 2019

More thoughts on reliability-based design -- Joe Murphy

- Short term strength distribution is independent of future applied loads, thus the summing of variances in FOSM is statistically correct.
- The ASD load-time factors have consensus time spans for the primary load in specific load combinations for the 5th-percentile. Say 1 month for snow load.
- The ASD safety format can be considered a special case of the LRFD safety format with the Load and Resistance Factors all equal to 1.
- The wood community has calculated reliability index using AFOSM, excluding load-time factors, getting $\beta_3 = 2.4$.
- The steel community calculates reliability index using FOSM (not AFOSM) getting $\beta_2 = 3.0$. β_2 expanded (page 6) is:

$$\beta_2 = \frac{\ln(\bar{R} / \bar{Q})}{\sqrt{CV_R^2 + CV_Q^2}} = \frac{\ln(\bar{R}) - \ln(\bar{D} + \bar{X})}{\sqrt{\frac{V_R}{\bar{R}^2} + \frac{V_D + V_X}{(\bar{D} + \bar{X})^2}}}$$

- The reliability index using FOSM β_1 expanded (page 6) is:

$$\beta_1 = \frac{\bar{R} - \bar{Q}}{\sqrt{V_R + V_Q}} = \frac{\bar{R} - (\bar{D} + \bar{X})}{\sqrt{V_R + (V_D + V_X)}}$$

β_1 calculation is simple, using only basic distribution statistics and the requirement that applied loads are independent from short-term strength. Pages 23-26.

- β_1 and β_2 calculations are straight forward equations and algebra.
- For large \bar{R} , $\beta_2 > \beta_1$ because \bar{R} not only increases the numerator but also decreases the denominator in the β_2 equation.
- The difference in β_4 and β_5 is due to the skewness of the g distribution from the Monte Carlo simulation. The more negative the skewness the more $\beta_4 < \beta_5$
- The live load combination is being considered as the Standard load case with a target reliability index $\beta = 3$.
- Calculation of the format conversion factor, K_F , can be simply ratio-ing the safety checking equations of the different formats.
- Format conversion does not change reliability.
- AFOSM provides a simple way to develop the reliability normalization factor, K_R , using the "Design" option in NBS577js.html The "Design" option basically does multiple "Analysis"es until the program converges to the target reliability index. Developing K_R would be more difficult using Monte Carlo simulations.
- Different resistance factors ϕ_s s express different reliability indices which may be warranted for different stress modes.
- Constant reliability (index) for all stress modes requires the same resistance factor ϕ_s for all stress modes in reliability-based LRFD.
- Include load-time factors when doing AFOSM or other reliability analyses.

RELATION OF STRENGTH OF WOOD TO DURATION OF LOAD

December 1951



No. R1916

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
FOREST PRODUCTS LABORATORY
Madison 5, Wisconsin
In Cooperation with the University of Wisconsin

RELATION OF STRENGTH OF WOOD TO DURATION OF LOAD

By

LYMAN W. WOOD, Engineer

Forest Products Laboratory,¹ Forest Service
U. S. Department of Agriculture

Summary

This report presents, for the use of structural engineers, a mathematical expression for an important structural property of wood, the relation of its bending strength to the duration of load. The relationship is mathematically defined, and applications in working-stress problems are shown. Structural designers are thus enabled to take full advantage of the unusually high bending strength of wood under short-time loading.

Data showing the relationship are from recently completed tests of small, clear Douglas-fir beams under long-time load and from earlier studies of rapid loading and impact. An empirical hyperbolic equation is developed to represent the trends of the data. A few exploratory tests of other species and in other strength properties indicate that the relationship may be of general application.

Introduction

Designing engineers customarily set working stresses for structural materials at levels below the yield point, or elastic limit, to insure safe and satisfactory structures under service loading. They recognize that materials loaded beyond the elastic limit may lose elasticity and take on characteristics of brittleness or plasticity, according to their nature. It is less generally known that materials differ widely in this respect, and that in some the strength properties are greatly affected by the duration of loading. Wood has a property valuable to the structural designer in that both its elastic limit and its ultimate strength are higher under short-time than under long-time loading; this permits higher working stresses where live loads of comparatively short duration must be considered in structural design,

¹Maintained at Madison, Wis. , in cooperation with the University of Wisconsin.

The relation of the strength of wood to the duration of load has been investigated at the Forest Products Laboratory for many years. Recent analysis of duration data from a series of long-time loading tests of small, clear Douglas-fir beams led to a restudy of the relation of duration of load to the strength of wood and a revision of conclusions previously held. This paper summarizes the data considered in the restudy and reports the conclusions reached.

Data considered here are from bending tests of Douglas-fir at two levels of moisture content, both in the air-dry range. The conclusions cannot yet be extended to other species and strength properties or to wood in the green condition, although a few exploratory tests indicate that bending and other strength properties in some other species are similarly affected. Any general application of the conclusions is subject to revision as more complete information is obtained.

Sources of Data

A series of 126 long-time loading tests of 1- by 1-inch, clear Douglas-fir beams at 6 and 12 percent moisture content was begun in 1943 and is now approaching completion. Test specimens were subjected to constant loads ranging from 60 to 95 percent of the loads that caused failure of matched control specimens in a standard static-bending test of about 5 minutes' duration. Durations until failure under these loads ranged from a few minutes to more than 5 years. Figure 1 shows durations plotted against stress levels corresponding to the applied loads in each of the tests. Duration values are shown on a logarithmic scale,

studies of the effects of rapid rates of loading on small, clear Douglas-fir beams were reported by Liska². Data from that report and a line showing their trend are plotted in figure 2. The time scale is logarithmic. The data of figure 1 and figure 2 are not exactly comparable, since figure 1 shows durations of stress increasing to predetermined levels and then held constant, while figure 2 shows times of loading continuously increasing at a constant rate until failure. Nevertheless, it is believed that both sets of data are governed by the same properties of the material and should be represented by one continuous curve.

Impact tests, in which the actual forces or loads were observed, were reported by Elmendorf³. The data indicated that the modulus of rupture of Douglas-fir is about 75 percent greater in impact than in static bending as shown by the standard 5-minute test. Elmendorf's data on Douglas-fir

²Liska, J. A. Effect of Rapid Loading on the Compressive and Flexural Strength of Wood., Forest Products Laboratory Rept. No. R1767. 1950.

³Elmendorf, Armin. Stresses in Impact. Journal of the Franklin Institute. Vol. 182. No. 6, 1916.

did not show durations of stress but included results on one specimen of southern yellow pine for which the duration in the same kind of impact test was about 0.015 second, This duration, while not directly connected to the 175-percent strength level, indicates the high stresses that can be developed under extremely rapid loading.

Relation of Duration of Load to Strength

In figure 1, a straight line drawn by eye represents the trend of the data in the long-time loading tests. This line was published in a report by Wood⁴ and has since appeared with a modified scale of stress percentages in several other publications. Figure 2 shows a straight line representing the trend of the data from rapid-loading tests. The two straight lines and a single point representing Elmendorf's impact data are converted to the same scale and plotted together in figure 3. Here also the duration scale is logarithmic.

It is evident in figure 3 that both the rapid-loading and the impact data lie above the extension of the straight line representing the long-time loading data, and that the impact point lies above the extension of the line representing rapid loading. A curve representing all of these data cannot be a straight line.

The straight line of figure 1 implies that strength values decrease without limit as the duration is prolonged. This is obviously impossible, since strength cannot have a value less than zero. Experiments with wood in this and other countries indicate that there is probably a threshold strength level somewhere above zero for which the duration is infinite.

From this evidence, it appears that the over-all strength-duration relationship could well be approximated by a hyperbolic curve. The horizontal asymptote of such a curve would represent a threshold strength for which duration is infinite. The vertical asymptote would be at zero **time**, though there is admittedly no proof that the strength becomes infinite as zero time is approached.

Several attempts at curve-fitting showed that a hyperbola that represented the trends of data in long-time loading and rapid loading could not be passed through the point representing the impact loading. Each of these trends *is supported by many tests*, while the impact point is related to only one test. A curve was therefore chosen to fit the trends of the two large groups of tests and to pass as closely as possible to the impact point,

⁴Wood L.W. Behavior of Wood under Continued Loading. Engineering News-Record, Vol. 139, No. 24, 1947.

Figure 3 shows an empirical hyperbolic curve passing through a point representing a duration of stress of 0.015 second and a stress equal to 150 percent of ultimate strength in a standard test. This curve approximates the trends of data from the long-time-loading and the rapid-loading tests. It has the equation

$$y = \frac{108.4}{x^{0.04635}} + 18.3$$

in which x is the duration of stress in seconds and y is the stress expressed as a percentage of the standard-test strength. This equation is computed so that the curve passes through three selected points. The first point is the point just described and is somewhat below the impact point, The second point is at the 100-percent strength level, for which a duration of stress of 7-1/2 minutes was assumed. The third point is arbitrarily selected from the long-time loading data with a strength level of 69 percent and a duration of 3,750 hours (shown on fig. 1). The horizontal asymptote of this hyperbola is 18.3 percent, a strength level for which the duration is presumed to be infinite.

The hyperbolic curve of figure 3 is also shown on figure 1. It is well within the range of the long-time loading data, though toward the upper side of that range for strength levels of 65 percent and lower. On the other hand, a similar curve for bending strength of Sitka spruce⁵ shows still higher strength levels for this range of durations (fig. 1). The departure of the hyperbola from the general trend of data at the 95-percent strength level is necessary to fit it to the rapid-loading data of figure 2. In the absence of similar information from other species, this hyperbola may be taken to express a general relationship between strength and duration of load for those species most used in construction.

Application to Working Stresses

The relation of duration of load to strength is important in the determination of working stresses for structural design with wood. Advantage may be taken of the increased strength of wood under short-time loading by increasing the working stresses where maximum load is of limited duration. Figure 4 illustrates a convenient means for doing this. It shows the hyperbolic curve of figure 3 plotted in a form directly applicable to working-stress use. Basic working stresses recommended by the Forest Products Laboratory are for the condition of long-time full load, for which the strength is assumed to be nine-sixteenths of the strength in the standard 5-minute test. This long-time load level is taken as 100 percent in figure 4 with other percentages calculated from it as a base. Duration values are converted to units of time that are easily visualized for long as well as short durations.

⁵Forest Products Laboratory. Strength and Related properties of Wood Grown in the United States. U. S. Dept. of Agr. Tech. Bull. No. 479. 99 pp. Illus.

In the "National Design Specification for Stress-grade Lumber and Its Fastenings," revised 1950, recommended by the National Lumber Manufacturers Association for application to permanent structures, a condition designated as "normal loading" is selected as the basis for working stresses in structural design. The same condition is assumed in the commercial grading rules for stress-graded lumber, "Normal loading" contemplates that the full maximum design load will have a continuous or cumulative duration of not more than about 10 years during the life of a permanent structure. It will be seen in figure 4 that 10-year duration warrants an increase of about 10 percent above the long-time load level. The National Design Specification gives working stresses that contain the 10 percent increase with provision for removing that increase in cases where the full maximum load is applied permanently or for many years. That basis for working stresses conforms to the principle of adjustment for duration of load.

Maximum working stresses for roof structures in certain areas may be based on expected snow loads. For example, the duration of the greatest expected snow load in temperate climates may be considered by the designer to be only a matter of days, weeks, or at the most a few months during the expected life of the structure. Figure 4 indicates that an increase of about 25 percent over the long-time load level can be made in this instance. This is equivalent to an increase of about 15 percent above the "normal-loading" level.

In like manner, a designer may wish to assume that the maximum horizontal load, as from wind or earthquake, will have a duration not exceeding a matter of minutes or hours during the expected life of the structure. For this condition, a working stress may be 50 percent above the long-time load level or about one-third above the "normal-loading" level.

When applying design stress increases for short-time loading, care should be taken that the sizes of structural members are adequate for the dead or long-time portion of load at a safe long-time working stress. This is accomplished by comparing the working stresses required for a member at each load level and its expected duration. The larger of the working stresses governs the design of that member.

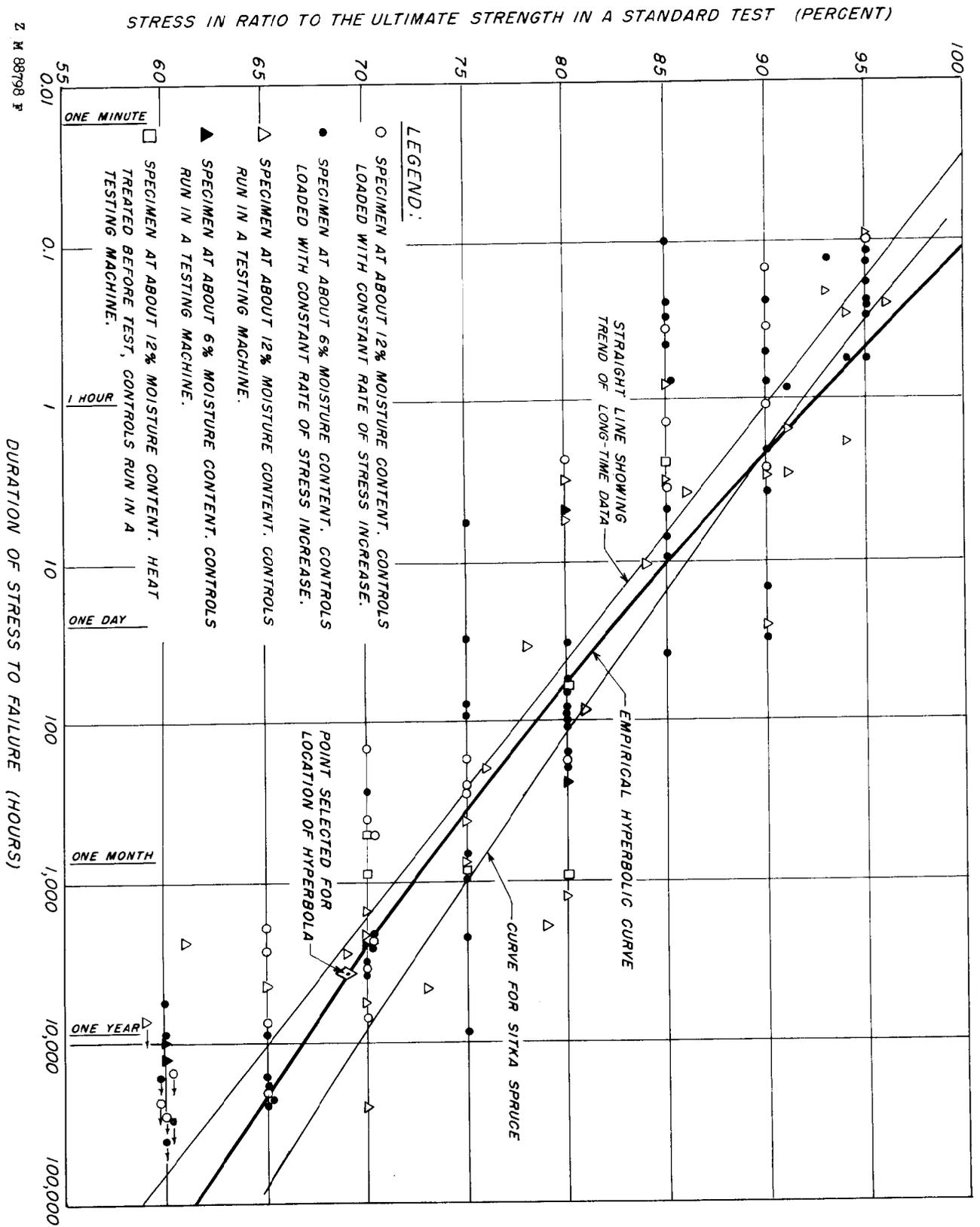


Figure 1.--Relation of duration of constant stress to level of stress in long-time loading of Douglas-fir bending specimens.

Z M 89381 P

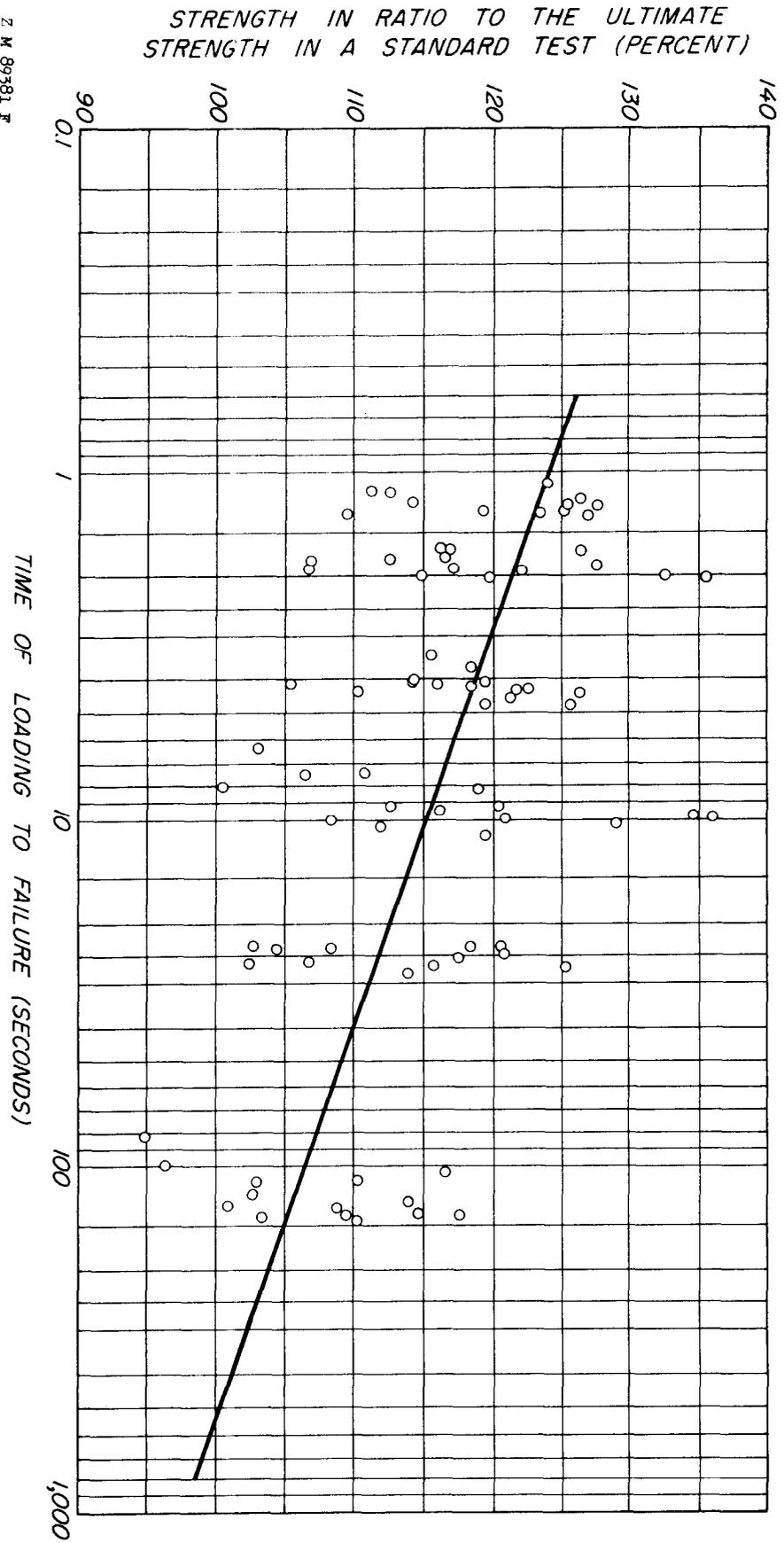
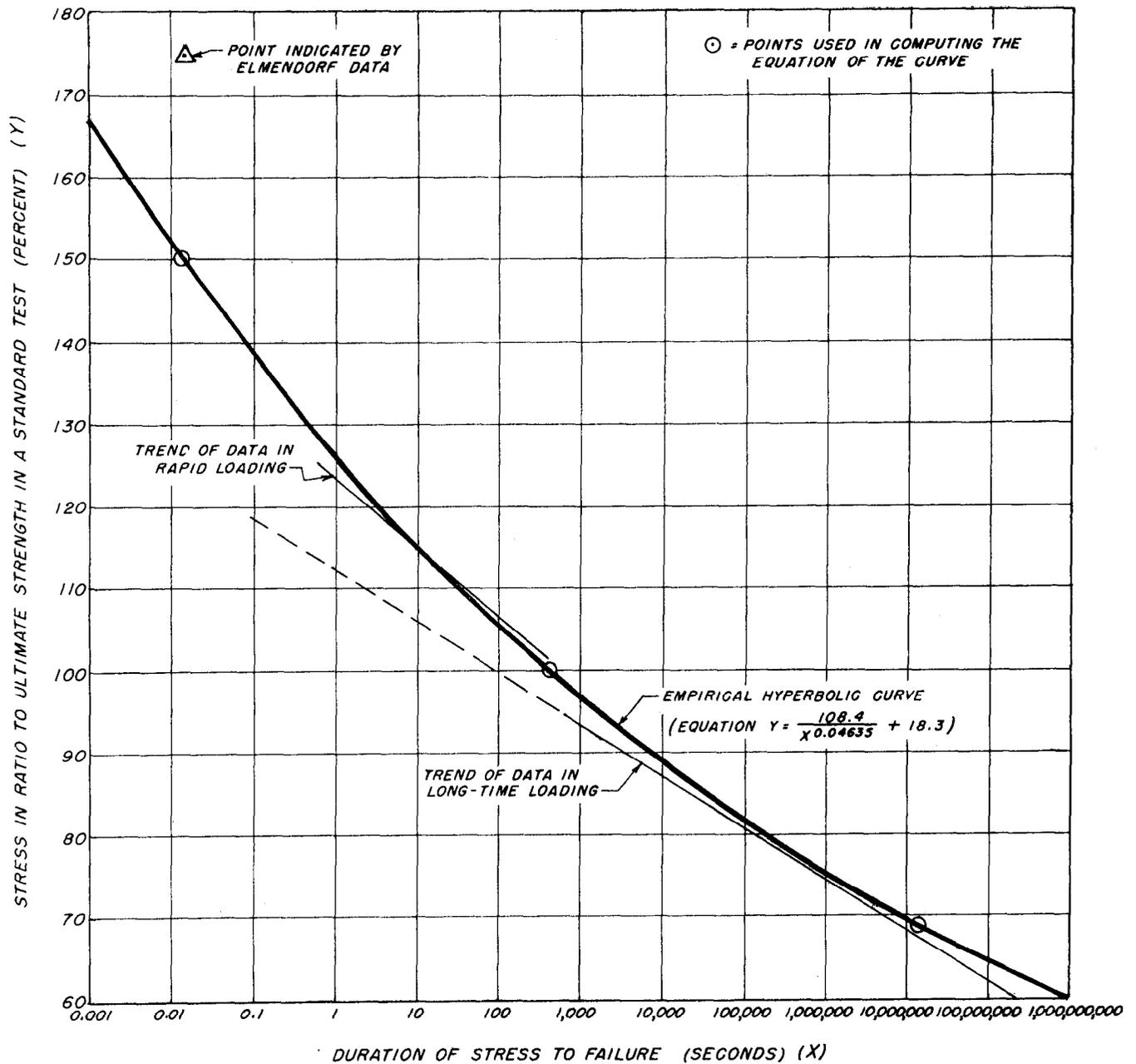


Figure 2.--Relation of strength to time of loading in rapid-loading tests of small, clear Douglas-fir bending specimens.



Z M 88799 F

Figure 3.--Relation of duration of stress to level of stress in long-time loading and rapid loading of Douglas-fir bending specimens. The broken line represents an extension of the curve for long-time loading.

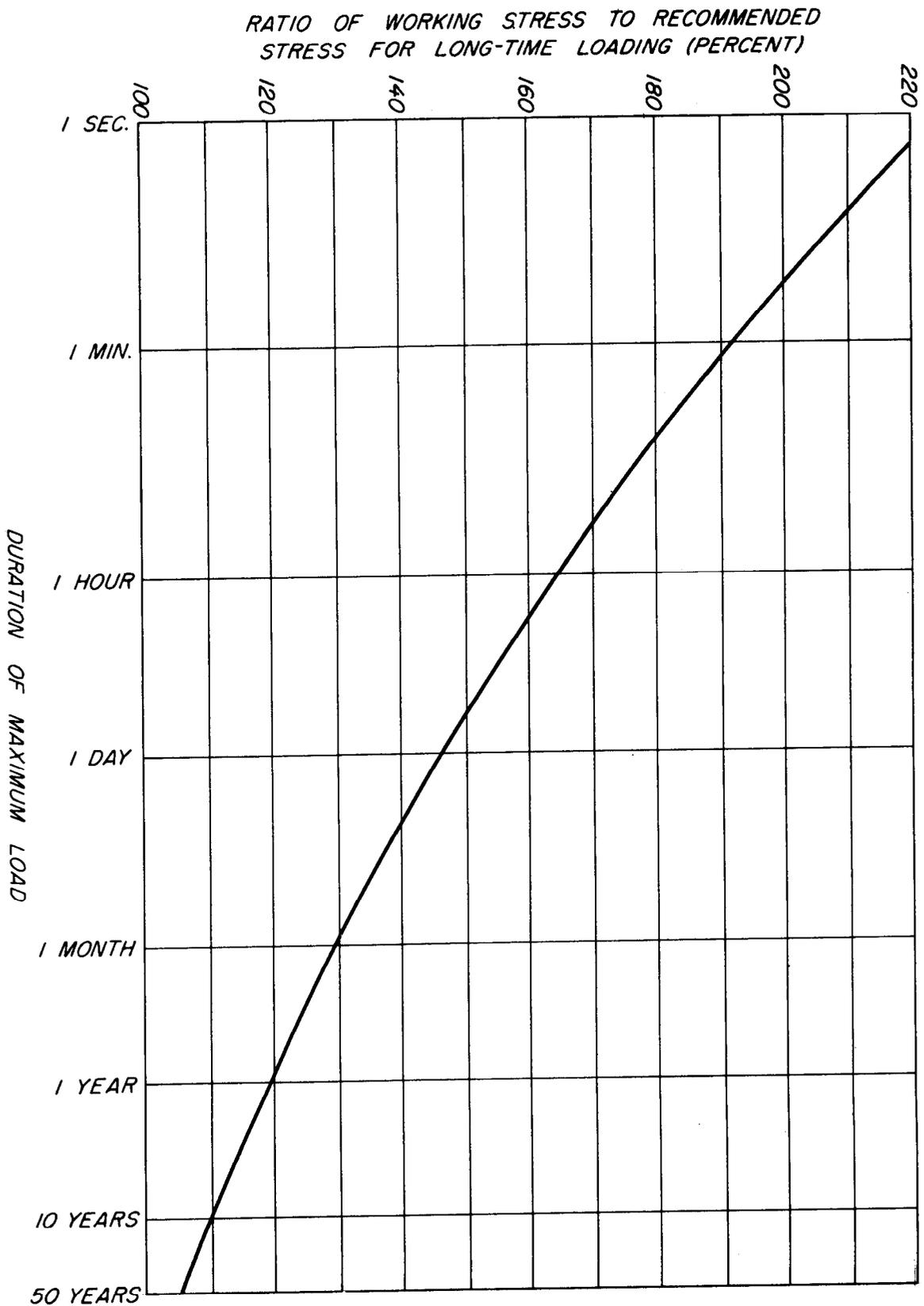


Figure 4.--Relation of working stress to duration of load.

Introductory Probability and Statistical Applications

PAUL L. MEYER

Department of Mathematics, Washington State University



ADDISON-WESLEY PUBLISHING COMPANY

Reading, Massachusetts · Palo Alto · London · Don Mills, Ontario

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Library of Congress Catalog Card No. 65-14525

Second Printing—November 1966

Property 7.2. Suppose that C is a constant and X is a random variable. Then $E(CX) = CE(X)$.

$$\text{Proof: } E(CX) = \int_{-\infty}^{+\infty} Cxf(x) dx = C \int_{-\infty}^{+\infty} xf(x) dx = CE(X).$$

Property 7.3. Let (X, Y) be a two-dimensional random variable with a joint probability distribution. Let $Z = H_1(X, Y)$ and $W = H_2(X, Y)$. Then $E(Z + W) = E(Z) + E(W)$.

Proof

$$\begin{aligned} E(Z + W) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [H_1(x, y) + H_2(x, y)]f(x, y) dx dy \\ &\quad \text{[where } f \text{ is the joint pdf of } (X, Y)\text{]} \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H_1(x, y)f(x, y) dx dy + \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H_2(x, y)f(x, y) dx dy \\ &= E(Z) + E(W). \end{aligned}$$

Property 7.4. Let X and Y be any two random variables. Then $E(X + Y) = E(X) + E(Y)$.

Proof: This follows immediately from Property 7.3 by letting $H_1(X, Y) = X$, and $H_2(X, Y) = Y$.

Note: Combining Properties 7.1, 7.2, and 7.4 we observe the following important fact: If $Y = aX + b$, where a and b are constants, then $E(Y) = aE(X) + b$. In words: The expectation of a linear function is that same linear function of the expectation. This is *not* true unless a linear function is involved, and it is a common error to believe otherwise. For instance, $E(X^2) \neq (E(X))^2$, $E(\ln X) \neq \ln E(X)$, etc. Thus if X assumes the values -1 and $+1$, each with probability $\frac{1}{2}$, then $E(X) = 0$. However,

$$E(X^2) = (-1)^2\left(\frac{1}{2}\right) + (1)^2\left(\frac{1}{2}\right) = 1 \neq 0^2.$$

Property 7.5. Let X_1, \dots, X_n be n random variables. Then

$$E(X_1 + \dots + X_n) = E(X_1) + \dots + E(X_n).$$

Proof: This follows immediately from Property 7.4 by applying mathematical induction.

Note: Combining this property with the above, we obtain

$$E\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n a_i E(X_i),$$

where the a_i 's are constants.

7.6 Properties of the Variance of a Random Variable

There are various important properties, in part analogous to those discussed for the expectation of a random variable, which hold for the variance.

Property 7.7. If C is a constant,

$$V(X + C) = V(X). \quad (7.13)$$

Proof

$$\begin{aligned} V(X + C) &= E[(X + C) - E(X + C)]^2 = E[(X + C) - E(X) - C]^2 \\ &= E[X - E(X)]^2 = V(X). \end{aligned}$$

Note: This property is intuitively clear, for adding a constant to an outcome X does not change its variability, which is what the variance measures. It simply “shifts” the values of X to the right or to the left, depending on the sign of C .

Property 7.8. If C is a constant,

$$V(CX) = C^2V(X). \quad (7.14)$$

Proof

$$\begin{aligned} V(CX) &= E(CX)^2 - (E(CX))^2 = C^2E(X^2) - C^2(E(X))^2 \\ &= C^2[E(X^2) - (E(X))^2] = C^2V(X). \end{aligned}$$

Property 7.9. If (X, Y) is a two-dimensional random variable, and if X and Y are independent then

$$V(X + Y) = V(X) + V(Y). \quad (7.15)$$

Proof

$$\begin{aligned} V(X + Y) &= E(X + Y)^2 - (E(X + Y))^2 \\ &= E(X^2 + 2XY + Y^2) - (E(X))^2 - 2E(X)E(Y) - (E(Y))^2 \\ &= E(X^2) - (E(X))^2 + E(Y^2) - (E(Y))^2 = V(X) + V(Y). \end{aligned}$$

Note: It is important to realize that the variance is *not additive*, in general, as is the expected value. With the additional assumption of independence, Property 7.9 is valid. Nor does the variance possess the linearity property which we discussed for the expectation, that is, $V(aX + b) \neq aV(X) + b$. Instead we have $V(aX + b) = a^2V(X)$.

Property 7.10. Let X_1, \dots, X_n be n independent random variables. Then

$$V(X_1 + \dots + X_n) = V(X_1) + \dots + V(X_n). \quad (7.16)$$

Proof: This follows from Property 7.9 by mathematical induction.



grain, and modulus of elasticity. In particular, the 1997 edition of the NDS includes the most up-to-date design values based on test results from an eight-year full-scale testing program that uses lumber samples from mills across the United States and Canada.

Characteristic structural properties for use in allowable stress design (ASTM D1990) and load and resistance factor design (ASTM D5457) are used to establish design values (ASTM, 1998a; ASTM, 1998b). Test data collected in accordance with the applicable standards determine a characteristic strength value for each grade and species of lumber. The value is usually the mean (average) or fifth percentile test value. The fifth percentile represents the value that 95 percent of the sampled members exceeded. In ASD, characteristic structural values are multiplied by the reduction factors in Table 5.1. The reduction factors are implicit in the allowable values published in the NDS-S for standardized conditions. The reduction factor normalizes the lumber properties to a standard set of conditions related to load duration, moisture content, and other factors. It also includes a safety adjustment if applicable to the particular limit state (i.e., ultimate capacity). Therefore, for specific design conditions that differ from the standard basis, design property values should be adjusted as described in Section 5.2.4.

The reduction factors in Table 5.1 are derived as follows as reported in ASTM D2915 (ASTM, 1997):

- F_b reduction factor = (10/16 load duration factor)(10/13 safety factor);
- F_t reduction factor = (10/16 load duration factor)(10/13 safety factor);
- F_v reduction factor = (10/16 load duration factor)(4/9 stress concentration factor) (8/9 safety factor);
- F_c reduction factor = (2/3 load duration factor)(4/5 safety factor); and
- $F_{c\perp}$ reduction factor = (2/3 end position factor)

5.2.4 Adjustment Factors

The allowable values published in the NDS-S are determined for a standard set of conditions. Yet, given the many variations in the characteristics of wood that affect the material's structural properties, several adjustment factors are available to modify the published values. For efficient design, it is important to use the appropriate adjustments for conditions that vary from those used to derive the standard design values. Table 5.2 presents adjustment factors that apply to different structural properties of wood. The following sections briefly discuss the adjustment factors most commonly used in residential applications. For information on other adjustment factors, refer to the NDS, NDS-S, and the NDS commentary.



TABLE 5.1 *Design Properties and Associated Reduction Factors for ASD*

Stress Property	Reduction Factor	Basis of Estimated Characteristic Value from Test Data	Limit State	ASTM Designation
Extreme fiber stress in bending, F_b	$\frac{1}{2.1}$	Fifth percentile	Ultimate capacity	D1990
Tension parallel to grain, F_t	$\frac{1}{2.1}$	Fifth percentile	Ultimate capacity	D1990
Shear parallel to grain, F_v	$\frac{1}{4.1}$	Fifth percentile	Ultimate capacity	D245
Compression parallel to grain, F_c	$\frac{1}{1.9}$	Fifth percentile	Ultimate capacity	D1990
Compression perpendicular to grain, $F_{c\perp}$	$\frac{1}{1.5}$	Mean	0.04" deflection ¹	D245
Modulus of elasticity, E	$\frac{1}{1.0}$	Mean	Proportional limit ²	D1990

Sources: ASTM, 1998a; ASTM, 1998c.

Notes:

¹The characteristic design value for $F_{c\perp}$ is controlled by a deformation limit state. In fact, the lumber will densify and carry an increasing load as it is compressed.

²The proportional limit of wood load-deformation behavior is not clearly defined because it is nonlinear. Therefore, designation of a proportional limit is subject to variations in interpretation of test data.

TABLE 5.2 *Adjustment Factor Applicability to Design Values for Wood*

Design Properties ¹	Adjustment Factor ²														
	C_D	C_r	C_H	C_F	C_P	C_L	C_M	C_{fu}	C_b	C_T	C_V	C_i	C_j	C_c	C_f
F_b	✓	✓		✓		✓	✓	✓			✓	✓	✓	✓	✓
F_t	✓			✓			✓					✓	✓		
F_v	✓		✓				✓					✓	✓		
$F_{c\perp}$							✓		✓			✓	✓		
F_c	✓			✓	✓		✓					✓	✓		
E							✓			✓		✓	✓		

Source: Based on NDS•2.3 (AF&PA, 1997).

Notes:

¹Basic or unadjusted values for design properties of wood are found in NDS-S. See Table 5.1 for definitions of design properties.

²Shaded cells represent factors most commonly used in residential applications; other factors may apply to special conditions.

Key to Adjustment Factors:

- C_D , Load Duration Factor. Applies when loads are other than "normal" 10-year duration (see Section 5.2.4.1 and NDS•2.3.2).
- C_r , Repetitive Member Factor. Applies to bending members in assemblies with multiple members spaced at maximum 24 inches on center (see Section 5.2.4.2 and NDS•4.3.4).



- C_H , Horizontal Shear Factor. Applies to individual or multiple members with regard to horizontal, parallel-to-grain splitting (see Section 5.2.4.3 and NDS-S).
- C_F , Size Factor. Applies to member sizes/grades other than "standard" test specimens, but does not apply to Southern Yellow Pine (see Section 5.2.4.4 and NDS-S).
- C_P , Column Stability Factor. Applies to lateral support condition of compression members (see Section 5.2.4.5 and NDS•3.7.1).
- C_L , Beam Stability Factor. Applies to bending members not subject to continuous lateral support on the compression edge (see Section 5.2.4.6 and NDS•3.3.3).
- C_M , Wet Service Factor. Applies where the moisture content is expected to exceed 19 percent for extended periods (see NDS-S).
- C_{fu} , Flat Use Factor. Applies where dimension lumber 2 to 4 inches thick is subject to a bending load in its weak axis direction (see NDS-S).
- C_b , Bearing Area Factor. Applies to members with bearing less than 6 inches and not nearer than 3 inches from the members' ends (see NDS•2.3.10).
- C_T , Buckling Stiffness Factor. Applies only to maximum 2x4 dimension lumber in the top chord of wood trusses that are subjected to combined flexure and axial compression (see NDS•4.4.3).
- C_V , Volume Factor. Applies to glulam bending members loaded perpendicular to the wide face of the laminations in strong axis bending (see NDS•5.3.2).
- C_t , Temperature Factor. Applies where temperatures exceed 100°F for long periods; not normally required when wood members are subjected to intermittent higher temperatures such as in roof structures (see NDS•2.4.3 and NDS•Appendix C).
- C_i , Incising Factor. Applies where structural sawn lumber is incised to increase penetration of preservatives with small incisions cut parallel to the grain (see NDS•2.3.11).
- C_c , Curvature Factor. Applies only to curved portions of glued laminated bending members (see NDS•5.3.4).
- C_f , Form Factor. Applies where bending members are either round or square with diagonal loading (see NDS•2.3.8).

5.2.4.1 Load Duration Factor (C_D)

Lumber strength is affected by the cumulative duration of maximum variable loads experienced during the life of the structure. In other words, strength is affected by both the load intensity and its duration (i.e., the load history). Because of its natural composition, wood is better able to resist higher short-term loads (i.e., transient live loads or impact loads) than long-term loads (i.e., dead loads and sustained live loads). Under impact loading, wood can resist about twice as much stress as the standard 10-year load duration (i.e., "normal duration") to which wood bending stress properties are normalized in the NDS.

When other loads with different duration characteristics are considered, it is necessary to modify certain tabulated stresses by a load duration factor (C_D) as shown in Table 5.3. Values of the load duration factor, C_D , for various load types



are based on the total accumulated time effects of a given type of load during the useful life of a structure. C_D increases with decreasing load duration.

Where more than one load type is specified in a design analysis, the load duration factor associated with the shortest duration load is applied to the entire combination of loads. For example, for the load combination, *Dead Load + Snow Load + Wind Load*, the load duration factor, C_D , is equal to 1.6.

TABLE 5.3 *Recommended Load Duration Factors for ASD*

Load Type	Load Duration	Recommended C_D Value
Permanent (dead load)	Lifetime	0.9
Normal	Ten years	1.0
Occupancy (live load) ¹	Ten years to seven days	1.0 to 1.25
Snow ²	One month to seven days	1.15 to 1.25
Temporary construction	Seven days	1.25
Wind and seismic ³	Ten minutes to one minute	1.6 to 1.8
Impact	One second	2.0

Source: Based on NDS•2.3.2 and NDS•Appendix B (AF&PA, 1997).

Notes:

¹The NDS uses a live load duration of ten years ($C_D = 1.0$). The factor of 1.25 is consistent with the time effect factor for live load used in the new wood LRFD provisions (AF&PA, 1996a).

²The NDS uses a snow load duration of one month ($C_D = 1.15$). The factor of 1.25 is consistent with the time effect factor for snow load used in the new wood LRFD provisions (AF&PA, 1996a).

³The NDS uses a wind and seismic load duration of ten minutes ($C_D = 1.6$). The factor may be as high as 1.8 for earthquake loads which generally have a duration of less than 1 minute with a much shorter duration for ground motions in the design level range.

5.2.4.2 Repetitive Member Factor (C_r)

When three or more parallel dimension lumber members are spaced a maximum of 24 inches on center and connected with structural sheathing, they comprise a structural “system” with more bending capacity than the sum of the single members acting individually. Therefore, most elements in a house structure benefit from an adjustment for the system strength effects inherent in repetitive members.

The tabulated design values given in the NDS are based on single members; thus, an increase in allowable stress is permitted in order to account for repetitive members. While the NDS recommends a repetitive member factor of 1.15 or a 15 percent increase in bending strength, system assembly tests have demonstrated that the NDS repetitive member factor is conservative for certain conditions. In fact, test results from several studies support the range of repetitive member factors shown in Table 5.4 for certain design applications. As shown in Table 5.2, the adjustment factor applies only to extreme fiber in bending, F_b . Later sections of Chapter 5 cover other system adjustments related to concentrated loads, header framing assemblies, and deflection (stiffness) considerations.