To: The Record

From: Joseph F. Murphy, Ph.D., Retired Research Engineer, Retired P.E.

Subject: Allowable Stress Design, ASD, vs Load and Resistance Factor Design, LRFD, Checking Equations. A Mathematical Comparison of Reliability

## Foreward

I am writing this technical report to record in one place my efforts and observations in understanding reliability math and how it is applied to wood. The views I share might be the same as others or not. This is not a typical report in that it is not directed at anyone but is more for a historical record for my own use and for anyone who might be interested.

I started programming computers in 1967 and enjoy (yes, you might find that hard to believe, but I do enjoy) math and solving math problems with computers. It should then come as no surprise that solving reliability math problems with a computer is something I like to do, even in retirement.

Some things that you should know. I'm writing this report using a text editor and the HTML (Hyper Text Markup Language) language. The equations use a MathJax javascript extension. I open the *.html file in a browser and "print" it to a pdf (Portable Document Format) file. I combine different *.pdf files using a pdftk (pdf tool kit). I developed the graphs using the postscipt (ps) programming language and distilled the postscript *.ps files into *.pdf files. Because the graphs were written in postscript one can zoom in those pdf file up to, I think, 600 times magnification (without distortion!) I finally "stamped" the page numbers with the date on the lower right corner using the pdftk.

If when reading the report in Adobe Reader and you are connected to the internet, the underlined hyperlinks should work, if I wrote them correctly and the website/page is still active.

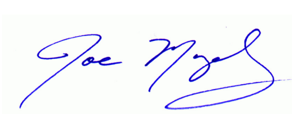
Also while in Adobe Reader (in Windows) I use the following shortcuts to jump around a *.pdf file and switch back and forth between portrait and landscape modes:

- Ctrl-Shift-'N' = Go To Page
- Ctrl-Shift-'+' = rotate view cw ↻ | Ctrl-Shift-'-' rotate view ccw ↺
- Alt-'⟵' = go to previous view | Alt-'⟶' go to subsequent view

I refer in the report to my page numbers and not original page numbers. I reference page numbers with double capital letters, e.g., AA BB CC etc. I wrote a BASIC (Beginner's All-purpose Symbolic Instruction Code) program to replace the double cap letters with real numbers. Hopefully, I got them all.

The References are what I used to write this report and should not be construed as being all inclusive. This report might look similar to a formal technical report but I assure you it is not.

I hope you find this report useful. Now let's get technical.



## Background

After spending ten years at Northwestern University, earning degrees in Civil Engineering, Biomedical Engineering, and Theoretical and Applied Mechanics, I started work at the USDA Forest Service Forest Products Laboratory (FPL) in the Engineering Design Criteria Research Work Unit in 1976. After a few years I became Project Leader of the Load Duration Design Criteria Research Work Unit. I was active is the ASCE (American Society of Civil Engineers) and the Committee on Wood of the Structural Division of the ASCE. I chaired the ASCE Task Committee on Load and Resistance Factor Design (LRFD) for Engineered Wood Construction.

I was involved with the ASTM (founded as the American Society for Testing and Materials) D-5457 Standard Specification for Computing Reference Resistance of Wood-Based Materials and Structural

Connections for Load And Resistance Factor Design. For the ASCE task committee members I programmed and compiled BASIC programs (with my own Assembly language subroutines) to calculate reliability indices. (At that time PC's ran at 4.77 MHz and a math coprocessor had to be installed separately). For ASTM I helped write the sections of the standard about fitting data, with or without censoring, to a 2 parameter Weibull distribution. I also programmed and compiled a BASIC program to fit data sets in compliance with ASTM D-5457.

Around 1983 I had a year sabbatical at NASA in San Francisco. I left FPL in 1985 and started Structural Reliability Consultants. In 1988 ASCE published: Load and Resistance Factor Design for Engineered Wood Construction, A Pre-Standard Report, Prepared by the Task Committee on Load and Resistance Factor Design for Engineered Wood Construction. See pp 10 for the title page, abstract, preface, introduction, and task committee membership.

In 1991 I rejoined FPL and after about a year I became Group Leader of the Engineering Mechanics Laboratory (EML). EML does nearly all the physical testing at FPL, especially large wood components and large data sets. Around 1998 I went back to research concentrating on high wind loading and wood building roof response, including a research structure in the panhandle of Florida. My research led me to modify an air cannon at FPL to launch a 2x4, testing saferoom log walls. I retired from FPL in January 2011.

In 2014 Dave Gromala contacted me about updating the wood community's LRFD effort. Since I had kept nearly all my computer programs, correspondence, and drafts of the pre-standard, I agreed to help out. I am coauthor of 4 journal articles appearing in Wood Design Focus, Volume 27, Number 1, Spring 2017, see References section.

In the process of reaquainting myself with reliability analyses of wood, I wrote a new computer program pp 49 based on NBS Special Publication 577, see References section. The input is exactly the same as the NBS 577 Program pp 36. My output contains the same outout as NBS 577 but contains additional detailed output [in square brakets].

**Random Variables and How the Design Checking Equation Positions the Probability Density Function Distributions.**(No Reliability Calculations!)

First some definitions concerning a random variable, $X$. (All are taken from Wikipedia.)

- In probability theory, the **expected value** of a random variable, intuitively, is the long-run average value of repetitions of the experiment it represents. The law of large numbers states that the arithmetic mean of the values almost surely converges to the expected value as the number of repetitions approaches infinity. The expected value is also known as the expectation, mathematical expectation, EV, average, mean value, **mean**, or first moment. My notation is $E(X)$, $\overline{X}$ , or $\mu$.

- In probability theory and statistics, **variance** is the expectation of the squared deviation of a random variable from its mean, and it informally measures how far a set of (random) numbers are spread out from their mean. The variance is the square of the standard deviation, the second central moment of a distribution, and the covariance of the random variable with itself, and it is often represented by $\sigma^2$, $s^2$, or $Var(X)$.

- In statistics, the **standard deviation** (SD, also represented by $\sigma$ or $s$) is a measure that is used to quantify the amount of variation or dispersion of a set of data values. A low standard deviation indicates that the data points tend to be close to the mean (also called the expected value) of the set, while a high standard deviation indicates that the data points are spread out over a wider range of values.

  The standard deviation of a random variable, statistical population, data set, or probability distribution is the square root of its variance. A useful property of the standard deviation is that, unlike the variance, it is expressed in the same units as the data.

- In probability theory and statistics, the **coefficient of variation** (CV), also known as relative standard deviation (RSD), is a standardized measure of dispersion of a probability distribution or frequency distribution. It is often expressed as a percentage, and is defined as the ratio of the standard deviation $\sigma$ to the mean $\mu$, i.e., $\frac{\sigma}{\mu}$ or $\frac{\sqrt{Var(X)}}{E(X)}$. It is commonly used in fields such as engineering or physics when doing quality assurance studies. My notation is $CV(X)$.

What happens if we multiply a random variable, say $X$, by a constant, $c$, to get a new random variable $cX$?

$E(cX) = cE(X)$ or $\overline{cX} = c\overline{X}$

$Var(cX) = c^2 Var(X)$ and $\sigma_{cX} = c\sigma_X$

$$CV(cX) = \frac{\sqrt{Var(cX)}}{E(cX)} = \frac{\sqrt{c^2 Var(X)}}{cE(X)} = \frac{\sqrt{Var(X)}}{E(X)} = CV(X)$$

The coefficient of variation of a random variable times a constant is the same as the coefficient of variation of the random variable. Now what happens if we divide the mean value of a random variable, say $\overline{X}$, by a fractile of distribution of the random variable, and here I'm just suggesting the 5th percentile $X_{05}$ or $\frac{\overline{X}}{X_{05}}$? If we multiply the random variable $X$ by a constant $c$ the mean-to-fifth ratio is

$$\frac{\overline{cX}}{(cX)_{05}} = \frac{c\overline{X}}{cX_{05}} = \frac{\overline{X}}{X_{05}}$$

Therefore multiplying a random variable by a constant does not change its mean-to-fifth percentile ratio or its coefficient of variation. See p 14 for mean-to-fifth ratios for three distributions.

For the resistance random variable we consider all the variability or uncertainty to be the short-term stength distribution. We do ASTM bending tests but we should not take into account actual dimensions, because the end-user uses nominal dimensions, for example. Using span and tributary areas we would multiply the strength distribution by a constant but it would not change the mean-to-fifth ratio or the coefficient of variation. Mathematically we take any constant multiplier out.

Now let's turn our attention to the design checking equations. I'll use the snow load combination for an example.

In ASD we have the unfactored loads,

(1) $D_n + S_n \leq R_{ASD}$

In LRFD we have factored loads,

(2) $1.2\, D_n + 1.6\, S_n \leq \lambda\, \varphi\, R_n$

In LRFD the nominal resistance, $R_n$, is multiplied by a resistance factor, $\varphi$, and a time effect factor, $\lambda$. The time effect factor is strictly dependent on the load combination and the most important load in that combination. See p 15 for the number of load combinations. Equation (1) has no similar factor. I propose that the ASD design checking equation should be,

(3) $D_n + S_n \leq C_D\, R_{ASD}$

to make it comparable to LRFD. $C_D$ is a duration of load factor. I call $\lambda$ and $C_D$ load-time-model factors for they both accomplish the same purpose. These load-time-model factors are based on simulations using three models:

Maximum load distribution model

Time duration model corresponding to the load levels

Damage accumulation model which depends on stress ratio where strength is assumed known

None of the models has been standardized. Each of these models has its own uncertainty. Models for different loads (e.g., snow load and live load) are not combined. The physical tests that have been done have been under constant load on solid wood or small clear wood specimens, usually at high stress levels to get results in a reasonable time. One cannot prove the results in the field, only in a limited testing environment. There have not been any field reports of any wood products failing due to time only. The load-time-model factor is applied to all wood products, solid and engineered wood products (e.g., plywood, wood-I beams.) Load-time modelling affects only a very small fraction of the low tail of the resistance distribution. All the excellent research on loads, load duration, damage accumulation modeling, and computer simulations have guided thoughtful wood code writers to specify sliding load-time-model factors, from impact to permanent loading. Their collective judgment maintains successful wood design and experience.

These load-time-model factors are very different than the other end-use factors. See pp 16 Table 5.2 for the end-use factors, usually based on Student t-tests following ASTM Standards for testing. The other end-use factors are completely independent of load combinations or a particular load. These factors are dependent on resistance mode, e.g., bending, tension, compression, shear. The load-time-model factors are completely dependent on the load side of the equation and independent of resistance mode. It only makes sense to me that the load-time-model factors should be included when we specify loads. All the other end-use factors multiply the short-term strength distribution but do not change the mean-to-fifth ratio or CV of the end-use distribution.

Let's position some load distributions. First a word on the probability density function, pdf. (Not to be confused with Adobe's portable document format.) From Wikipedia

> In probability theory, a probability density function (PDF), or density of a continuous random variable, is a function. The PDF is used to specify the probability of the random variable falling within a particular range of values, as opposed to taking on any one value. This probability is given by the integral of this variable's PDF over that range—that is, it is given by the area under the density function but above the horizontal axis and between the lowest and greatest values of the range. The probability density function is nonnegative everywhere, and its integral over the entire space is equal to one.

From NBS SP 577 we can use the load distributions characteristics see p 20 Table 3.1.

Dead Load $\dfrac{\overline{D}}{D_n} = 1.05 \quad CV_D = 0.10$ Normal pdf

Snow Load $\dfrac{\overline{S}}{S_n} = 0.82 \quad CV_S = 0.26$ Frechet pdf

We can take $D_n = 1.0$ and $S_n = 3.0$ for a $\dfrac{S_n}{D_n} = 3.0$. We take $D_n = 1.0$ for the sake of convenience. We could take $D_n = 20$ psf and $S_n = 60$ psf for the same $\dfrac{S_n}{D_n}$ ratio but we could also multiply the random variables by $c = 0.05$ psf$^{-1}$ and the mean-to-nominal ratios and CV's would not change, just the scale of the horizontal axis. The relative spacing of the distributions is what is important. The pdfs of these two distributions are plotted on p 21. The left (green) pdf represents the Dead load variable and the right (red) pdf represents Snow load variable. The tic marks are where $D_n$ and $S_N$ are located. The lines to the top of the distributions are where $\overline{D}$ and $\overline{S}$ are located.

Now where to put the resistance pdf?

Using equation (3) for ASD we have on the RHS $C_D\, R_{ASD}$

We get $R_{ASD}$ from Table 5.1 pp 16 for different resistance modes. For bending $R_{ASD} = \dfrac{R_{05}}{2.1}$.

We get $C_D$ from Table 5.3 pp 16 for Snow loads and we'll use $C_D = 1.15$

$$C_D\, R_{ASD} = C_D\,\frac{R_{05}}{2.1} = \left(\frac{C_D}{1.6}\right)\left(\frac{1.6}{2.1}\right) R_{05} = \left(\frac{1.15}{1.6}\right)\left(\frac{1.6}{2.1}\right) R_{05} = 0.548\, R_{05}$$

Now the first factor is like $\lambda$ in LRFD and the second factor is like $\varphi$ in LRFD. See p 22 for a comparison of the load-time-model factors, $\lambda$ and $C_D$.

Using equation (2) for LRFD we have on the RHS $\lambda\,\varphi\,R_n$. From ASTM D-5457 we have a conversion factor $R_n = \left(\dfrac{2.16}{\varphi}\right) R_{ASD}$. We cancel out $\varphi$ in the equation because $R_{ASD}$ accounts for different resistance modes (bending, tension, shear, compression.) We'll use $\lambda = 0.8$.

$$\lambda\,\varphi\,R_n = \lambda\,\varphi\left(\frac{2.16}{\varphi}\right)R_{ASD} = \lambda\,2.16\left(\frac{R_{05}}{2.1}\right) = (0.8)\left(\frac{2.16}{2.1}\right)R_{05} = 0.823\,R_{05}$$

For $D_n = 1$, $S_n = 3$ and ASD, we have

$$D_n + S_n = 0.548\,R_{05} \quad \text{giving} \quad R_{05} = 7.299$$

For LRFD

$$1.2D_n + 1.6S_n = 0.823\,R_{05} \quad \text{giving} \quad R_{05} = 7.290$$

It doesn't matter what the resistance distribution is because it is the same in ASD and LRFD and with the $R_{05}$ value being the same, the positioning is the same. The dashed magenta LRFD distribution overlays the solid blue ASD distribution. The tic marks are respective $R_{05}$. (Zoom in to see how close they are.) The resistance distribution used was

Resistance $\dfrac{\overline{R}}{R_{05}} = 1.546 \quad CV_R = 0.20$ Weibull pdf

(See p 20 and pp 24 for 5 distributions used by NBS SP 577 for loads. Weibull, Type III Extreme Value distribution is included as an option for resistance. Calculating pdfs for six different distributions are shown in pp 24.)

Because ASD and LRFD position the resistance distribution the same, I looked at the ratio of $\dfrac{(R_{05})_{LRFD}}{(R_{05})_{ASD}}$ for different $\dfrac{S_n}{D_n}$ ratios.

| $S_n/D_n$ | 1 | 2 | 3 | 4 | 5 | 5 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $(R_{05})_{LRFD}/(R_{05})_{ASD}$ | 0.9317 | 0.9761 | 0.9983 | 1.0116 | 1.0204 | 1.0268 | 1.0315 | 1.0352 | 1.0382 |

For this snow load case, these ratios hold for all resistance modes since $R_{ASD}$ will appear in both design checking equations. When $(R_{05})_{LRFD}/(R_{05})_{ASD}$ is greater than 1, LRFD is more conservative than ASD and vice versa, but the difference is small over a range of load ratios.

For different resistance modes the resistance can move. For shear where $R_{ASD} = \dfrac{R_{05}}{4.1}$ the resistance distribution moves away from the load distributions and has increased reliability. For compression where $R_{ASD} = \dfrac{R_{05}}{1.9}$ the resistance distribution moves slightly closer to the load distributions and have decreased reliability. This assumes that shear and compression have the same Weibull distribution as the bending example I used.

I have shown in this section that equations (2) and (3) give identical results for both LRFD and ASD.

Now that we can position the load and resistance distributions how do we go about calculating reliability and a reliability index? That's for the next section.

**(Bonus Section Live Load Case)**

With the same (bending) design checking equations (2) and (3) we substitute live load for snow load.

Live Load $\dfrac{\overline{L}}{L_n} = 1.0 \quad CV_L = 0.25$ Gumbel pdf

We also use $C_D = 1.0$ and $\lambda = 0.7$ which gives for

ASD RHS $0.476\,R_{05}$ and with $D_n = 1, L_n = 3$ then $R_{05} = 8.400$

LRFD RHS $0.720\,R_{05}$ and with $D_n = 1, L_n = 3$ then $R_{05} = 8.333$

| $L_n/D_n$ | 1 | 2 | 3 | 4 | 5 | 5 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $(R_{05})_{LRFD}/(R_{05})_{ASD}$ | 0.9259 | 0.9700 | 0.9921 | 1.0053 | 1.0141 | 1.0204 | 1.0251 | 1.0288 | 1.0317 |

## Reliability Calculations

### FOSM First Order Second Moment Reliability Index

Reviewing the sum of random variables, we know the expected value of the sum of random variables is equal to the sum of the expected values of the random variables.

$$X_\Sigma = \Sigma\, X_i \text{ then } E(X_\Sigma) = \Sigma\,(E(X_i)) \text{ or } \overline{X}_\Sigma = \Sigma\,\overline{X}_i$$

We also know that the variance of the sum of random variables is equal to the sum of the variances of the random variables.

$$Var(X_\Sigma) = \Sigma\,(Var(X_i)) \text{ and } CV(X_\Sigma) = \frac{\sqrt{Var(X_\Sigma)}}{E(X_\Sigma)} = \frac{\sqrt{\Sigma\,(Var(X_i))}}{\Sigma\,\overline{X}_i}$$

For our example we define a new random variable, $Z$, as $Z = R - D - S$. We get

$$E(Z) = \overline{Z} = \overline{R} - \overline{D} - \overline{S} \text{ and } Var(Z) = Var(R) + Var(D) + Var(S)$$

and

(4) $$\frac{1}{CV(Z)} = \frac{\overline{R} - \overline{D} - \overline{S}}{\sqrt{Var(R) + Var(D) + Var(S)}} = \beta$$

This is the first order second moment FOSM definition of the reliability index $\beta$. As the coefficient of variation of $Z$ goes up, the reliability index $\beta$ goes down and vice versa. In manufacturing quality control you might have heard the phrase "six sigmas". Well, we can use the phrase "$\beta$ sigmas"! Here there is nothing said about any distributions, only the expected values and variances. (I wrote a program that reads the same input file as the one NBS SP 577 computer program uses and my program calculates the FOSM reliability index $\beta$.) The FOSM reliability index should be considered "notional" since it has no connection to any probability!

Here are $\beta$ results if we use the nominal values, $D_n$, $S_n$, and $R_{05}$ from the checking equation and the corresponding $\frac{\overline{D}}{D_n}, \frac{\overline{S}}{S_n},$ and $\frac{\overline{R}}{R_{05}}$ and knowing $Var(D) = \left(\overline{D} \times CV(D)\right)^2, Var(S) = \left(\overline{S} \times CV(S)\right)^2$, and $Var(R) = \left(\overline{R} \times CV(R)\right)^2$.

| $S_n/D_n$ | 1 | 2 | 3 | 4 | 5 | 5 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| ASD Snow FOSM $\beta$ | 3.27 | 3.30 | 3.31 | 3.32 | 3.32 | 3.32 | 3.32 | 3.32 | 3.32 |
| LRFD Snow FOSM $\beta$ | 3.14 | 3.26 | 3.31 | 3.34 | 3.35 | 3.36 | 3.37 | 3.38 | 3.38 |

### (Bonus Section Live Load Case)

Substituting live load for the snow load we get

| $L_n/D_n$ | 1 | 2 | 3 | 4 | 5 | 5 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| ASD Live FOSM $\beta$ | 3.35 | 3.32 | 3.30 | 3.29 | 3.28 | 3.27 | 3.27 | 3.26 | 3.26 |
| LRFD Live FOSM $\beta$ | 3.21 | 3.27 | 3.29 | 3.30 | 3.31 | 3.31 | 3.31 | 3.32 | 3.32 |

### AFOSM Advanced First Order Second Moment Reliability Index

The reliability index $\beta$ in equation (4) is true for all types of distributions. It *IS* related to probability *IF* all the distributions are Normal distributions. We can write equation (4) thusly

(5) $$\frac{1}{CV(Z^*)} = \frac{\overline{R}^* - \overline{D}^* - \overline{S}^*}{\sqrt{Var(R^*) + Var(D^*) + Var(S^*)}} = \beta^*$$

where the $^*$ will refer to random variables with Normal distributions.

Now we can state the probability as

$$Pr[R^* - D^* - S^* < 0] = \Phi( - \beta^*)$$

where $\Phi()$ is the cdf, cumulative distribution function of the Normal distribution.

The question is what normal distributions should we use in place of the non-normal distributions? At a specific point (design or checking point) of each distribution, we make the probability density function pdf and the cumulative distribution function cdf of a normal distribution "equivalent" to the pdf and cdf of the non-normal distribution.

Reliability calculations using Advanced First Order Second Moment AFOSM (both the NBS computer program pp 36 and my computer program pp 49) satisfy Equation (5) and the following 11 equations (can be expanded to 15 equations total if we add another load variable).

At (checking) points on the distributions, $r^*$, $d^*$, and $s^*$, we require

$$0 = r^* - d^* - s^*$$

$$\text{cdf}_{R^*}(r^*) = \text{cdf}_R(r^*) \quad \text{and} \quad \text{pdf}_{R^*}(r^*) = \text{pdf}_R(r^*)$$

$$\text{cdf}_{D^*}(d^*) = \text{cdf}_D(d^*) \quad \text{and} \quad \text{pdf}_{D^*}(d^*) = \text{pdf}_D(d^*)$$

$$\text{cdf}_{S^*}(s^*) = \text{cdf}_S(s^*) \quad \text{and} \quad \text{pdf}_{S^*}(s^*) = \text{pdf}_S(s^*)$$

$$r^* = \overline{R}^* + \alpha_{R^*} \beta^* \sqrt{Var(R^*)}$$

$$d^* = \overline{D}^* + \alpha_{D^*} \beta^* \sqrt{Var(D^*)}$$

$$s^* = \overline{S}^* + \alpha_{S^*} \beta^* \sqrt{Var(S^*)}$$

and $$1 = \alpha_{R^*}^2 + \alpha_{D^*}^2 + \alpha_{S^*}^2$$

Resistance has $\alpha_{R^*} < 0$ and loads have $\alpha_{D^*} > 0, \alpha_{S^*} > 0$. The $\alpha$'s are direction cosines and the $R - D - S = 0$ surface is linearized at the checking or design points. Searching for the direction cosines minmizes the reliability index $\beta^*$. Please see pages 16-24 of NBS SP 577 for the mathematical details.

Enough of what the program does. Here are some computer results.

**AFOSM Computer Results** and an observation

One could run a large number of inputs but I will record just a few. In the snow load case I looked at $S_n/D_n$ ratios from 1 to 9 with Weibull resistance of $\overline{R}/R_{05} = 1.645$ and $CV(R) = 0.20$ for both ASD, equation (3) and LRFD, equation (2). I also looked at the $S_n/D_n = 3.0$ with Weibull resistances varying from $CV(R) = 0.10$ to $0.30$ (see p 14 for $\overline{R}/R_{05}$ ratios) again for ASD and LRFD.

The reliability index versus $S_n/D_n$ is shown on p 31. The solid line is ASD, dashed cyan line is LRFD. Top curves are FOSM from equation (4).

The reliability index versus resistance $CV(R)$ is shown on p 32. The dashed cyan LRFD line is right on top of the solid ASD line. This is also the case for the upper FOSM lines. There seems to be something amiss at the lower variability AFOSM curves. My observation is that depending on starting value for $\beta$, convergent criteria, including converging on direction cosines, $\alpha$'s, or checking/design points, $x_i^*$'s, and/or convergent threshold, one can get slightly different answers. I think this is because of the thick upper tail of the Frechet distribution and thick lower tail of the Weibull distribution pp 24.

**Bonus Section Live Load Case**

I also looked at substituting live load for snow load with results on p 33 and p 34. They look very similar to the snow load case. When I graphed both cases they fell right on top of each other. I provide the postscript arrays used to plot the reliability index on p 35. One can compare the different cases and see that the $\beta$'s are close to each other.

### Recommendations

- The most important recommendation I propose is to put the duration of load factor $C_D$ in the unfactored load design checking equation (3) and take it out from the end-use factors.
- Use the FOSM equation (4) for calculating general reliability.

  It is simple to use.

  It does not depend on distributions.

  It is statistically correct, $\beta$ is "notional" not probability related.

  Do we really know loads, including the EUL "equivalent uniform load", and models at all calculating steps, to justify precision of stating the probability of something occurring or not? From my experience with LRFD input (e.g., precision, modeling, distribution fitting), math (e.g., linearizing limit states, normalizing non-normal distributions, design checking equations, load-time-model factor), and computer programming (e.g., convergent algorithm, convergent criteria, significant digits), I would definitely say NO. I recommend to use AFOSM with FOSM to guide code writers but do not claim anything more than a "notional" reliability index, being interpreted in a comparative sense, not a relative frequency sense.
- FOSM can incorporate quality assurance, NDT sorting, and proofloading easily. Removing low quality material from a distribution raises the mean and reduces the variance, both a help in the FOSM equation (4). It is a double benefit, easily documentable.

Well this has been a long and winding road. Time to wrap it up. A lot of good memories.

### References

pp 36 : APPENDIX F - COMPUTER PROGRAM from Ellingwood, B., Galambos, T.V., MacGregor, J.G., and Cornell, C.A. (1980). "Development of a Probability Based Load Criterion for American National Standard A58," Special Publication 577, National Bureau of Standards, U.S. Dept. of Commerce, Washington, D.C. (Digitized by Google) Written in mainframe Fortran (FORmula TRANslation) language. 222 pages.

pp 49 : The NBS SP 577 computer program ported over to PowerBASIC and compiled with PowerBASIC Console Compiler 6.01 for Windows. PowerBASIC, Inc. 1978 Tamiami Trail South, Venice, FL 34293, (941) 408-8700, http://www.powerbasic.com sp57764.exe can run on 32 or 64 bit Windows personal computers.

pp 10 : first 7 pages of Load and Resistance Factor Design for Engineered Wood Construction: A Pre-Standard Report by Task Committee on Load and Resistance Factor Design for Engineered Wood Construction, ASCE, American Society of Civil Engineers, New York, NY 978-0-87262-679-9 (ISBN-13) | 0-87262-679-2 (ISBN-10), 1988, Soft Cover, 177 pages.

Zahn. John J . 1992 . Fortran programs for reliability analysis. General Technical Report FPL-GTR-72. Madison. WI: U.S. Department of Agriculture. Forest Service. Forest Products Laboratory. 25 pages.

ASTM D5457, Standard Specification for Computing Reference Resistance of Wood-Based Materials and Structural Connections for Load and Resistance Factor Design, ASTM International, West Conshohocken, PA, 2015, http://www.astm.org

Wood Design Focus, A Journal of Contemporary Wood Engineering, Volume 27, Number 1, Spring 2017. Issue Theme: Load and Resistance Factor Design – Update for Wood

- Keeping Pace with Evolving LRFD Terminology in Design Standards -- David S. Gromala, PE, Philip Line, PE, Joseph F. Murphy, PhD, Thang Dao, PhD
- Do Design Adjustment Factors for Wood Affect the Computed Reliability Index: And If Not, Why Not? -- David S. Gromala, PE, Philip Line, PE, Joseph F. Murphy, PhD, Kevin C.K. Cheung, PhD

- Example Calculations to Derive Input Values for Reliability Analysis of Wood Products -- David S. Gromala, PE, Philip Line, PE, Joseph F. Murphy, PhD, Thang Dao, PhD
- Historical Approach Making a Comeback: Closed Form Equations to Determine LRFD Reliability Indices ($\beta$) and Resistance Factors ($\varphi$) -- David S. Gromala, PE, Philip Line, PE, Joseph F. Murphy, PhD, Thang Dao, PhD

**More material is available**

I have more material available which I have uploaded to my website. You can access the website at http://joeinmadison.com/lrfd You will find these files.

- readme1.pdf - a copy of this section
- jfmreport.pdf - a copy of this report
- sp57764.zip - a zip files containing source *.bas code and *.jpg files. The *jpg files are just *.exe files with the wrong extension name. One must rename the *.jpg files with a *.exe extension for them to function. (I store them as *.jpg to pass through some file transfer protocols.) All the *.bas source code is compiled into *.exe executables with PowerBASIC Console Compiler 6.01 for Windows. Also contains input *.txt files for sp57764.exe which I used in this report.

  sp57764.bas program listing like pp 49, sp57764.jpg compiled with PowerBASIC
  input filename.txt (just like NBS 577 APPENDIX F pp 36), output filename.htm
  input filename.txt uses the DOS 8.3 file naming convention.

  NBSbeta.bas *.jpg program scans the sp57764 output for the beta values
  input filename.htm, output filenameNBS.dat

  FOSMbeta.bas *.jpg program to compute FOSM beta values using sp57764 input
  input filename.txt, output filenameFOSM.dat

  do1dot.bat batch file to do all input *.txt files in a folder
  do1dot.bat, sp57764.exe, NBSbeta.exe, FOSMbeta.exe have to be in the same folder

  do2dot.bat batch file to do all input *.txt files in a folder
  do2dot.bat has to be in the same folder
  sp57764.exe, NBSbeta.exe, FOSMbeta.exe have to be in one folder above (parent folder)

  snowS2D.txt - input *.txt file for sp57764.exe
  snowCV.txt - input *.txt file for sp57764.exe
  liveL2D.txt - input *.txt file for sp57764.exe
  liveCV.txt - input *.txt file for sp57764.exe

- NBSSP577.pdf - a copy of NBS Special Publication 577, National Bureau of Standards. Digitized by Google.
- fplgtr72.pdf - John Zahn's General Technical Report with his Fortran program listing. Some of the code contains typos. Referenced by ASTM D-5457. See References.
- WDF_Q1_2017.pdf - Wood Design Focus, A Journal of Contemporary Wood Engineering, Volume 27, Number 1, Spring 2017. See References.

Programs I have written can be used for personal use only!
*.exe files in the *.zip file have been renamed with a *.jpg
extension. Rename them back to *.exe to be executable.

You can reach me at joe_@mailbag.com

# Load and Resistance Factor Design for Engineered Wood Construction

Joe Murphy

## A Pre-Standard Report

Prepared by the Task Committee on Load and Resistance Factor Design for Engineered Wood Construction of the Committee on Wood of the Structural Division of the American Society of Civil Engineers

**Joseph F. Murphy, Chairman**

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New York, New York 10017-2398

## ABSTRACT

The information contained in this pre-standard report is intended to provide guidance to writers of a load and resistance factor design (LRFD) specification for engineered wood construction. The report contains procedures and a methodology to determine resistance factors for forest products used in engineered wood construction. The resistance factors were developed using concepts of probabilistic limit states design which incorporated state-of-the-art load and resistance models and statistical information, and they are consistent with the rationale used in determining load factors and load combinations in ANSI A58—Building Code Requirements for Minimum Design Loads in Buildings and Other Structures. The phenomena of load duration (creep-rupture) was research and a load-duration factor, λ, was developed. This load duration factor incorporates a stochastic load process model as well as a damage accumulation model. Resistance and design equations, including buckling and ponding, are formulated in terms of load capacities. Fire and serviceability limit states are discussed. Additional factors which account for quality control, data sample size, and numerous forest products with different variabilities, are also presented.



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## Preface

The chapters and appendices of this pre-standard were written by people who shared a desire to take the first step in developing a reliability-based load and resistance factor design (LRFD) specification for the engineering design of single elements of wood construction. The intended users of this pre-standard are specification writers, researchers, teachers and technical personnel of the wood industry. For the LRFD specification user, an example LRFD safety checking equation, for snow load and dead load, would look like:

$$\lambda \phi R_n \geq 1.6 S_n + 1.2 D_n$$

where the 1.6 and 1.2 are load factors on nominal snow and dead load effects, respectively. $\phi$ is a resistance factor on the element's nominal resistance capacity $R_n$ and λ is an additional factor to account for load-duration phenomena. This LRFD pre-standard gives guidance and procedures for determining $\phi$ and λ so as to assure uniform reliability of forest products across a given range of applications. The writers were given broad guidelines and a few common analytical techniques and were asked to address their chapters independently. Detailed LRFD background information is provided in the first two chapters. Readers should completely understand the material in these first two chapters before studying the remaining chapters and appendices. Issues such as data sample size, choice of population distributional form, and target reliability indices, $\beta_o$, were not addressed in detail. Load duration is addressed in the second chapter.

When this effort was initiated, it was realized that accounting for load duration (or creep rupture) was an important task that had to be researched. We knew the load-duration phenomena hasn't changed, but perhaps our perception of it might. The task of developing a load-duration factor, λ, was assigned to the second chapter authors. In all the other chapters and appendices, reliability indices, β, and resistance factors, $\phi$, are based on five-minute short-term strength distributions and maximum load distributions just like steel and reinforced concrete. (If load duration is accounted for the β's are lower.) These β's and $\phi$'s can be used for comparison of wood products but NOT in comparison with steel or concrete. The use of the 5-minute resistance distribution in calculating the reliability index of present design treats the old load-duration factor and ASTM factor strictly as a safety factor (e.g., 1.15/2.1 = 1/1.83 for the snow load combination of a bending element.)

New standards are needed. As shown in chapter three, when a lognormal distribution was fitted to seven sample data sets, the reliability indices calculated were higher than if a Weibull distribution was fitted to the same sample data sets. Discussions about the relative merits of various assumed distributional forms were common during the development of the pre-standard. For standards development, a 'baseline' distribution form would provide consistency. Most steel and reinforced concrete reliability calculations use assumed distributions (i.e., concrete uses a normal, steel uses a lognormal.) Another significant issue is the fitting of parameters to the data. Fitting parameters to the lower tail (i.e. suspending data above a certain value) rather than the complete distribution adds more weight to the lower values. This can result in higher reliability but a LARGE unbiased data sample size is necessary to fit parameters using low tail values.

Conversion to LRFD is possible. However, in specific areas (noted in the chapters) such as connecting elements, the calibration will be coarse at best due to the paucity of ultimate strength data. Though the pre-standard does not address structural modeling, it is anticipated that good structural models for components such as trusses and glulam, and systems models for redundant systems such as light-frame floor, wall, and roof systems, will provide resistance distributions to use in reliability calculations. These diverse components and assemblies can be considered in an LRFD format with appropriate modeling uncertainty factors.

The writers believe that the information in this pre-standard provides useful guidance for those who will develop the LRFD specification for wood. However, experience and present satisfactory performance cannot be discounted. The calculated reliability of existing members in structures provides guidance as to the target reliability $\beta_o$ that should be designed into future members. This essentially calibrates future wood design to the current design process, which has a history of satisfactory performance.

## Introduction

Structural codes are evolving from a deterministic allowable stress design (ASD) basis to a limit state design basis in which the safety checks are derived from the underlying probability distributions of strength and loads. These changes will have a significant impact on the forest products industry and on the design of engineered wood construction.

At the October, 1983 National Science Foundation Workshop on Structural Wood Research in Milwaukee, it was recommended unanimously that the American Society of Civil Engineers' Committee on Wood undertake, as a pre-standardization activity, the development of a reliability-based design document. Subsequently, the ASCE Committee on Wood voted to form a Task Committee on Load and Resistance Factor Design (LRFD) for Engineered Wood Construction. The Task Committee initiated its work in October, 1984.

The development and implementation of LRFD for wood structures presents features that have not been encountered with reinforced concrete and steel construction. Wood is a natural material with nonuniform mechanical properties and growth characteristics. The strength of wood is dependent on the rate and duration of load, which makes the strength of wood structures dependent on load history. Moreover, the strength and stiffness distributions of forest products depend on the grading procedure (visual or mechanical), on the quality control in manufacture and on the wood species. Wood strength can be highly variable in comparison with the strengths observed for steel and reinforced concrete. Subsequent sections of this report will show how these special features in wood construction can be addressed in LRFD.

The Task Committee has prepared this resource document to: 1) explain why the adoption of limit states design is desirable for wood construction; 2) describe the analysis methods and data needed to determine load and resistance factors; and 3) show how practical safety checking design procedures are derived from this information. Each section gives an example code requirement as might be written by a standards-writing group and an extended commentary to explain the requirement, where appropriate.

The appendices show: how design for fire resistance might be formulated in a reliability framework; how to quantify benefits from improved quality control and screening procedures, and; how to develop a serviceability index for a serviceability limit state.

- Task Committee Chairman -

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Resistance Distributions

**Lognormal**

$$\mu = \ln\left(\overline{X}\right) - \frac{1}{2}\ln\left(1 + \frac{Var(X)}{\overline{X}^2}\right)$$

$$\sigma^2 = \ln\left(1 + \frac{Var(X)}{\overline{X}^2}\right) \quad \text{and} \quad (CV(X))^2 = \frac{Var(X)}{\overline{X}^2}$$

$$X_{05} = \exp(\mu - 1.645\,\sigma)$$

with $t = \ln\left[1 + (CV(X))^2\right]$ then $\frac{\overline{X}}{X_{05}} = \exp\left(\frac{1}{2}t + 1.645\sqrt{t}\right)$

**Normal**

$$\frac{\overline{X}}{X_{05}} = \frac{1}{1 - 1.645\,CV(X)}$$

**Weibull** where $\Gamma()$ is the Gamma function

$$\overline{X} = \eta\,\Gamma\left(1 + \frac{1}{\alpha}\right)$$

$$Var(X) = \eta^2\left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \left(\Gamma\left(1 + \frac{1}{\alpha}\right)\right)^2\right]$$

$(CV(X))^2 = \frac{Var(X)}{\overline{X}^2}$ Note: $\frac{1}{\alpha}$ is a function of $CV(X)$ only!

$X_p = \eta\,(-\ln(1-p))^{\frac{1}{\alpha}}$ and $\frac{\overline{X}}{X_{05}} = \frac{\Gamma\left(1 + \frac{1}{\alpha}\right)}{(-\ln(0.95))^{\frac{1}{\alpha}}}$

**Results** $\overline{X}/X_{05}$

| CoV | L | N | W |
|---|---|---|---|
| 0.10 | 1.184 | 1.197 | 1.224 |
| 0.11 | 1.205 | 1.221 | 1.251 |
| 0.12 | 1.226 | 1.246 | 1.279 |
| 0.13 | 1.248 | 1.272 | 1.309 |
| 0.14 | 1.270 | 1.299 | 1.339 |
| 0.15 | 1.292 | 1.328 | 1.370 |
| 0.16 | 1.315 | 1.357 | 1.403 |
| 0.17 | 1.339 | 1.388 | 1.437 |
| 0.18 | 1.363 | 1.421 | 1.472 |
| 0.19 | 1.388 | 1.455 | 1.508 |
| 0.20 | 1.413 | 1.490 | 1.546 |
| 0.21 | 1.438 | 1.528 | 1.585 |
| 0.22 | 1.464 | 1.567 | 1.625 |
| 0.23 | 1.491 | 1.609 | 1.667 |
| 0.24 | 1.518 | 1.652 | 1.711 |
| 0.25 | 1.545 | 1.699 | 1.756 |
| 0.26 | 1.574 | 1.747 | 1.803 |
| 0.27 | 1.602 | 1.799 | 1.851 |
| 0.28 | 1.632 | 1.854 | 1.902 |
| 0.29 | 1.662 | 1.912 | 1.954 |
| 0.30 | 1.692 | 1.974 | 2.008 |

**TABLE 3.1** ***Typical Load Combinations Used for the Design of Components and Systems[1]***

| Component or System | ASD Load Combinations | LRFD Load Combinations |
|---|---|---|
| Foundation wall (gravity and soil lateral loads) | D + H<br>D + H + $L^2$ + 0.3($L_r$ + S)<br>D + H + ($L_r$ or S) + 0.3$L^2$ | 1.2D + 1.6H<br>1.2D + 1.6H + 1.6$L^2$ + 0.5($L_r$ + S)<br>1.2D + 1.6H + 1.6($L_r$ or S) + 0.5$L^2$ |
| Headers, girders, joists, interior load-bearing walls and columns, footings (gravity loads) | D + $L^2$ + 0.3 ($L_r$ or S)<br>D + ($L_r$ or S) + 0.3 $L^2$ | 1.2D + 1.6$L^2$ + 0.5 ($L_r$ or S)<br>1.2D + 1.6($L_r$ or S) + 0.5 $L^2$ |
| Exterior load-bearing walls and columns (gravity and transverse lateral load) [3] | Same as immediately above plus<br>D + W<br>D + 0.7E + 0.5$L^2$ + 0.2$S^4$ | Same as immediately above plus<br>1.2D + 1.5W<br>1.2D + 1.0E + 0.5$L^2$ + 0.2$S^4$ |
| Roof rafters, trusses, and beams; roof and wall sheathing (gravity and wind loads) | D + ($L_r$ or S)<br>0.6D + $W_u{}^5$<br>D + W | 1.2D + 1.6($L_r$ or S)<br>0.9D + 1.5$W_u{}^5$<br>1.2D + 1.5W |
| Floor diaphragms and shear walls (in-plane lateral and overturning loads) [6] | 0.6D + (W or 0.7E) | 0.9D + (1.5W or 1.0E) |

Notes:

[1]The load combinations and factors are intended to apply to nominal design loads defined as follows: D = estimated mean dead weight of the construction; H = design lateral pressure for soil condition/type; L = design floor live load; $L_r$ = maximum roof live load anticipated from construction/maintenance; W = design wind load; S = design roof snow load; and E = design earthquake load. The design or nominal loads should be determined in accordance with this chapter.

[2]Attic loads may be included in the floor live load, but a 10 psf attic load is typically used only to size ceiling joists adequately for access purposes. However, if the attic is intended for storage, the attic live load (or some portion) should also be considered for the design of other elements in the load path.

[3]The transverse wind load for stud design is based on a localized component and cladding wind pressure; D + W provides an adequate and simple design check representative of worst-case combined axial and transverse loading. Axial forces from snow loads and roof live loads should usually not be considered simultaneously with an extreme wind load because they are mutually exclusive on residential sloped roofs. Further, in most areas of the United States, design winds are produced by either hurricanes or thunderstorms; therefore, these wind events and snow are mutually exclusive because they occur at different times of the year.

[4]For walls supporting heavy cladding loads (such as brick veneer), an analysis of earthquake lateral loads and combined axial loads should be considered. However, this load combination rarely governs the design of light-frame construction.

[5]$W_u$ is wind uplift load from negative (i.e., suction) pressures on the roof. Wind uplift loads must be resisted by continuous load path connections to the foundation or until offset by 0.6D.

[6]The 0.6 reduction factor on D is intended to apply to the calculation of net overturning stresses and forces. For wind, the analysis of overturning should also consider roof uplift forces unless a separate load path is designed to transfer those forces.

# 3.3 Dead Loads

Dead loads consist of the permanent construction material loads comprising the roof, floor, wall, and foundation systems, including claddings, finishes, and fixed equipment. The values for dead loads in Table 3.2 are for commonly used materials and constructions in light-frame residential buildings. Table 3.3 provides values for common material densities and may be useful in calculating dead loads more accurately. The design examples in Section 3.10 demonstrate the straight-forward process of calculating dead loads.

grain, and modulus of elasticity. In particular, the 1997 edition of the NDS includes the most up-to-date design values based on test results from an eight-year full-scale testing program that uses lumber samples from mills across the United States and Canada.

Characteristic structural properties for use in allowable stress design (ASTM D1990) and load and resistance factor design (ASTM D5457) are used to establish design values (ASTM, 1998a; ASTM, 1998b). Test data collected in accordance with the applicable standards determine a characteristic strength value for each grade and species of lumber. The value is usually the mean (average) or fifth percentile test value. The fifth percentile represents the value that 95 percent of the sampled members exceeded. In ASD, characteristic structural values are multiplied by the reduction factors in Table 5.1. The reduction factors are implicit in the allowable values published in the NDS-S for standardized conditions. The reduction factor normalizes the lumber properties to a standard set of conditions related to load duration, moisture content, and other factors. It also includes a safety adjustment if applicable to the particular limit state (i.e., ultimate capacity). Therefore, for specific design conditions that differ from the standard basis, design property values should be adjusted as described in Section 5.2.4.

The reduction factors in Table 5.1 are derived as follows as reported in ASTM D2915 (ASTM, 1997):

- $F_b$ reduction factor = (10/16 load duration factor)(10/13 safety factor);
- $F_t$ reduction factor = (10/16 load duration factor)(10/13 safety factor);
- $F_v$ reduction factor = (10/16 load duration factor)(4/9 stress concentration factor) (8/9 safety factor);
- $F_c$ reduction factor = (2/3 load duration factor)(4/5 safety factor); and
- $F_{c\perp}$ reduction factor = (2/3 end position factor)

## 5.2.4 Adjustment Factors

The allowable values published in the NDS-S are determined for a standard set of conditions. Yet, given the many variations in the characteristics of wood that affect the material's structural properties, several adjustment factors are available to modify the published values. For efficient design, it is important to use the appropriate adjustments for conditions that vary from those used to derive the standard design values. Table 5.2 presents adjustment factors that apply to different structural properties of wood. The following sections briefly discuss the adjustment factors most commonly used in residential applications. For information on other adjustment factors, refer to the NDS, NDS-S, and the NDS commentary.

## TABLE 5.1 *Design Properties and Associated Reduction Factors for ASD*

| Stress Property | Reduction Factor | Basis of Estimated Characteristic Value from Test Data | Limit State | ASTM Designation |
|---|---|---|---|---|
| Extreme fiber stress in bending, $F_b$ | $\frac{1}{2.1}$ | Fifth percentile | Ultimate capacity | D1990 |
| Tension parallel to grain, $F_t$ | $\frac{1}{2.1}$ | Fifth percentile | Ultimate capacity | D1990 |
| Shear parallel to grain, $F_v$ | $\frac{1}{4.1}$ | Fifth percentile | Ultimate capacity | D245 |
| Compression parallel to grain, $F_c$ | $\frac{1}{1.9}$ | Fifth percentile | Ultimate capacity | D1990 |
| Compression perpendicular to grain, $F_{c\perp}$ | $\frac{1}{1.5}$ | Mean | 0.04” deflection[1] | D245 |
| Modulus of elasticity, $E$ | $\frac{1}{1.0}$ | Mean | Proportional limit[2] | D1990 |

*Sources: ASTM, 1998a; ASTM, 1998c.*
Notes:
[1]The characteristic design value for $F_{c\perp}$ is controlled by a deformation limit state. In fact, the lumber will densify and carry an increasing load as it is compressed.
[2]The proportional limit of wood load-deformation behavior is not clearly defined because it is nonlinear. Therefore, designation of a proportional limit is subject to variations in interpretation of test data.

## TABLE 5.2 *Adjustment Factor Applicability to Design Values for Wood*

| Design Properties[1] | Adjustment Factor[2] | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $C_D$ | $C_r$ | $C_H$ | $C_F$ | $C_P$ | $C_L$ | $C_M$ | $C_{fu}$ | $C_b$ | $C_T$ | $C_V$ | $C_t$ | $C_i$ | $C_c$ | $C_f$ |
| $F_b$ | ✓ | ✓ | | ✓ | | ✓ | ✓ | ✓ | | | ✓ | ✓ | ✓ | ✓ | ✓ |
| $F_t$ | ✓ | | | ✓ | | | ✓ | | | | | ✓ | ✓ | | |
| $F_v$ | ✓ | | ✓ | | | | ✓ | | | | | ✓ | ✓ | | |
| $F_{c\perp}$ | | | | | | | ✓ | | ✓ | | | ✓ | ✓ | | |
| $F_c$ | ✓ | | | ✓ | ✓ | | ✓ | | | | | ✓ | ✓ | | |
| $E$ | | | | | | | ✓ | | | ✓ | | ✓ | ✓ | | |

*Source: Based on NDS•2.3 (AF&PA, 1997).*
Notes:
[1]Basic or unadjusted values for design properties of wood are found in NDS-S. See Table 5.1 for definitions of design properties.
[2]Shaded cells represent factors most commonly used in residential applications; other factors may apply to special conditions.

Key to Adjustment Factors:

- $C_D$, Load Duration Factor. Applies when loads are other than "normal" 10-year duration (see Section 5.2.4.1 and NDS•2.3.2).

- $C_r$, Repetitive Member Factor. Applies to bending members in assemblies with multiple members spaced at maximum 24 inches on center (see Section 5.2.4.2 and NDS•4.3.4).

- $C_H$, Horizontal Shear Factor. Applies to individual or multiple members with regard to horizontal, parallel-to-grain splitting (see Section 5.2.4.3 and NDS-S).
- $C_F$, Size Factor. Applies to member sizes/grades other than "standard" test specimens, but does not apply to Southern Yellow Pine (see Section 5.2.4.4 and NDS-S).
- $C_P$, Column Stability Factor. Applies to lateral support condition of compression members (see Section 5.2.4.5 and NDS•3.7.1).
- $C_L$, Beam Stability Factor. Applies to bending members not subject to continuous lateral support on the compression edge (see Section 5.2.4.6 and NDS•3.3.3).
- $C_M$, Wet Service Factor. Applies where the moisture content is expected to exceed 19 percent for extended periods (see NDS-S).
- $C_{fu}$, Flat Use Factor. Applies where dimension lumber 2 to 4 inches thick is subject to a bending load in its weak axis direction (see NDS-S).
- $C_b$, Bearing Area Factor. Applies to members with bearing less than 6 inches and not nearer than 3 inches from the members' ends (see NDS•2.3.10).
- $C_T$, Buckling Stiffness Factor. Applies only to maximum 2x4 dimension lumber in the top chord of wood trusses that are subjected to combined flexure and axial compression (see NDS•4.4.3).
- $C_V$, Volume Factor. Applies to glulam bending members loaded perpendicular to the wide face of the laminations in strong axis bending (see NDS•5.3.2).
- $C_t$, Temperature Factor. Applies where temperatures exceed 100$^o$F for long periods; not normally required when wood members are subjected to intermittent higher temperatures such as in roof structures (see NDS•2.4.3 and NDS•Appendix C).
- $C_i$, Incising Factor. Applies where structural sawn lumber is incised to increase penetration of preservatives with small incisions cut parallel to the grain (see NDS•2.3.11).
- $C_c$, Curvature Factor. Applies only to curved portions of glued laminated bending members (see NDS•5.3.4).
- $C_f$, Form Factor. Applies where bending members are either round or square with diagonal loading (see NDS•2.3.8).

### 5.2.4.1 Load Duration Factor ($C_D$)

Lumber strength is affected by the cumulative duration of maximum variable loads experienced during the life of the structure. In other words, strength is affected by both the load intensity and its duration (i.e., the load history). Because of its natural composition, wood is better able to resist higher short-term loads (i.e., transient live loads or impact loads) than long-term loads (i.e., dead loads and sustained live loads). Under impact loading, wood can resist about twice as much stress as the standard 10-year load duration (i.e., "normal duration") to which wood bending stress properties are normalized in the NDS.

When other loads with different duration characteristics are considered, it is necessary to modify certain tabulated stresses by a load duration factor ($C_D$) as shown in Table 5.3. Values of the load duration factor, $C_D$, for various load types are based on the total accumulated time effects of a given type of load during the useful life of a structure. $C_D$ increases with decreasing load duration.

Where more than one load type is specified in a design analysis, the load duration factor associated with the shortest duration load is applied to the entire combination of loads. For example, for the load combination, *Dead Load + Snow Load + Wind Load*, the load duration factor, $C_D$, is equal to 1.6.

**TABLE 5.3** ***Recommended Load Duration Factors for ASD***

| Load Type | Load Duration | Recommended $C_D$ Value |
|---|---|---|
| Permanent (dead load) | Lifetime | 0.9 |
| Normal | Ten years | 1.0 |
| Occupancy (live load)[1] | Ten years to seven days | 1.0 to 1.25 |
| Snow[2] | One month to seven days | 1.15 to 1.25 |
| Temporary construction | Seven days | 1.25 |
| Wind and seismic[3] | Ten minutes to one minute | 1.6 to 1.8 |
| Impact | One second | 2.0 |

*Source: Based on NDS•2.3.2 and NDS•Appendix B (AF&PA, 1997).*
Notes:
[1]The NDS uses a live load duration of ten years ($C_D$ = 1.0). The factor of 1.25 is consistent with the time effect factor for live load used in the new wood LRFD provisions (AF&AP, 1996a).
[2]The NDS uses a snow load duration of one month ($C_D$ = 1.15). The factor of 1.25 is consistent with the time effect factor for snow load used in the new wood LRFD provisions (AF&PA, 1996a).
[3]The NDS uses a wind and seismic load duration of ten minutes ($C_D$ = 1.6). The factor may be as high as 1.8 for earthquake loads which generally have a duration of less than 1 minute with a much shorter duration for ground motions in the design level range.

## 5.2.4.2 Repetitive Member Factor ($C_r$)

When three or more parallel dimension lumber members are spaced a maximum of 24 inches on center and connected with structural sheathing, they comprise a structural “system” with more bending capacity than the sum of the single members acting individually. Therefore, most elements in a house structure benefit from an adjustment for the system strength effects inherent in repetitive members.

The tabulated design values given in the NDS are based on single members; thus, an increase in allowable stress is permitted in order to account for repetitive members. While the NDS recommends a repetitive member factor of 1.15 or a 15 percent increase in bending strength, system assembly tests have demonstrated that the NDS repetitive member factor is conservative for certain conditions. In fact, test results from several studies support the range of repetitive member factors shown in Table 5.4 for certain design applications. As shown in Table 5.2, the adjustment factor applies only to extreme fiber in bending, $F_b$. Later sections of Chapter 5 cover other system adjustments related to concentrated loads, header framing assemblies, and deflection (stiffness) considerations.

values of the load effect are denoted through the subscripts "ann" and "apt." With the exception of E, the nominal loads are all defined by the values specified in the ANSI A58.1-1972 load standard. The nominal snow and wind are the 50-year mean recurrence interval values. The nominal earthquake load

Table 3.1 - Load Distributions and Parameters

| Load | $\bar{X}/X_n$ | $V_X$ | cdf |
|---|---|---|---|
| D | 1.05 | 0.10 | Normal |
| L | Eqs. 3.9 or 3.10 | 0.25 | Type I |
| $L_{apt}$ | Eq. 3.11 | Table A.2 | Gamma |
| W | 0.78 | 0.37 | Type I |
| $W_{ann}$ | 0.33 | 0.59 | Type I |
| $W_{apt}$ | (-0.021) | (18.7) | Type I |
| S | 0.82 | 0.26 | Type II |
| $S_{ann}$ | 0.20 | 0.73 | Lognormal |
| E | (Site dependent) Appendix A | (2.3) | Type II |

$E_n$ is the value from the 1976 edition of the Uniform Building Code. Values given in parentheses are characteristic extreme and shape parameters of extreme value distributions rather than mean and c.o.v. $V_X$ includes uncertainties due to inherent variability, load modeling and analysis.

Two values of the nominal live load $L_n$ are of interest in this study. The first is the value in ANSI A58.1-1972, which was used to determine the values of β which correspond to existing accepted practice. The corresponding $L_n$ is,

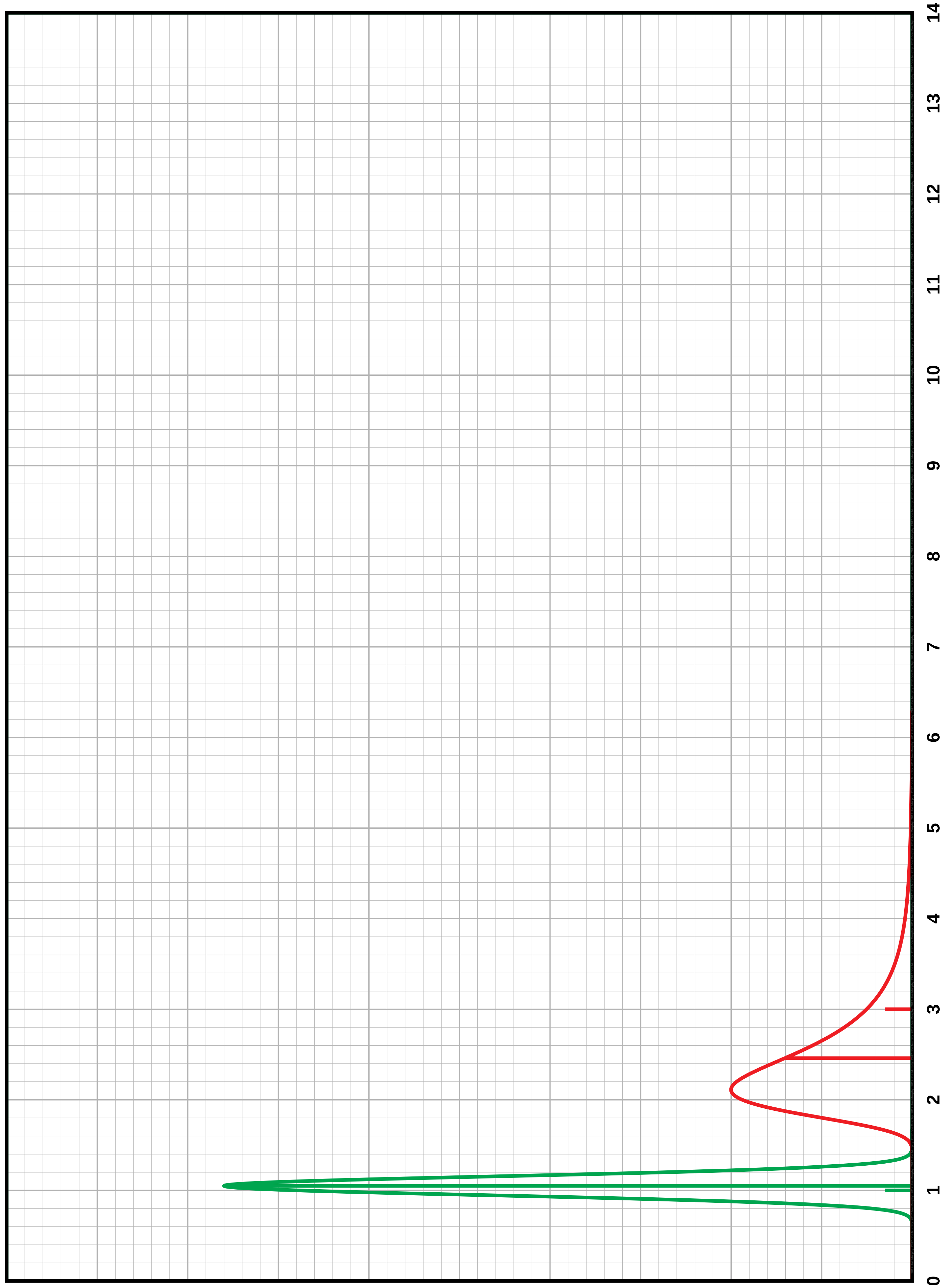
$$L_n = [1 - \min \{ 0.0008A_T, 0.6, 0.23(1 + \frac{D_n}{L_o})\}] L_o \qquad (3.9)$$

in which $A_T$ = tributary area (see glossary, Chapter 9) and $L_o$ = basic (unreduced) live load given in Table 1 of ANSI A58.1-1972. The second nominal live load is that proposed for the 1980 version of the A58 Standard,

$$L_n = [0.25 + 15/\sqrt{A_I}] L_o \qquad (3.10)$$

in which $A_I$ = influence area. This nominal value happens to equal the 50-year mean value, $\bar{L}$. The live load factor in the new load criterion is derived so as to be compatible with the 1980 nominal live load. Similarly, for the arbitrary point-in-time live load,

$$\bar{L}/L_n = \frac{0.24}{1 - \min \{0.0008A_T, 0.6, 0.23(1 + \frac{D_n}{L_0})\}} \quad \text{(A58.1-1972 Standard)} \qquad (3.11a)$$


0
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0
1
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3
4
5

**Load-time-model Factors**

**LRFD time effect factor, $\lambda$**

Table 2: Time Effect Factors based on Duration

| TimeFrame | $\lambda$ | Example |
|---|---|---|
| Greater than 10 years | 0.6 | Permanent |
| 10 years | 0.7 | Normal (Floor LL) |
| 2 months | 0.8 | Snow Load |
| 10 minutes | 1.0 | Wind or Seismic Load |
| Less than 2 seconds | 1.25 | Impact Load |

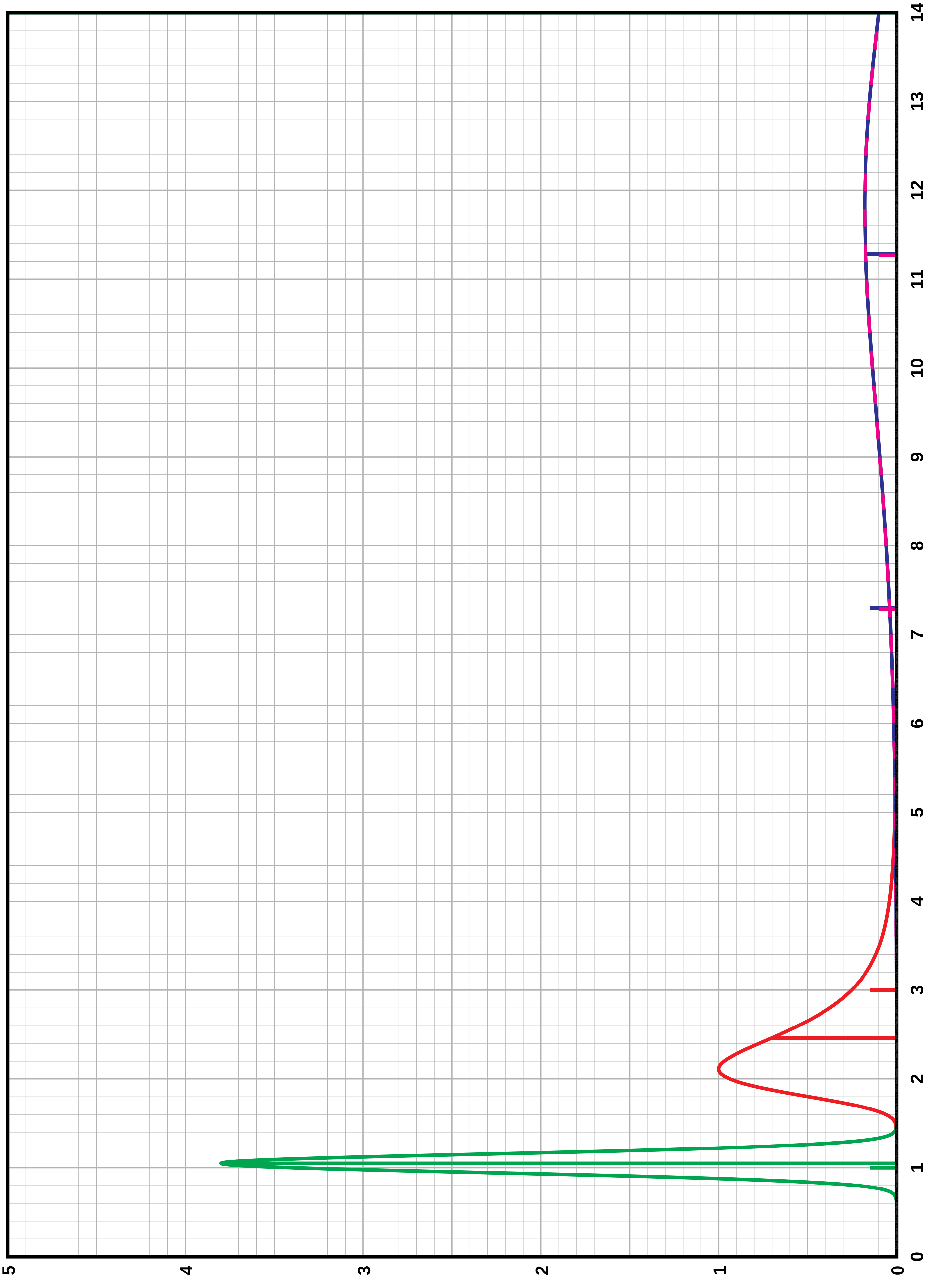
References: American Forest and Paper Association, "National Design Specification for Wood Construction", 2005 Found in Appendix N.

**ASD Load Duration Factor, $C_D$**

TABLE 5.3 Recommended Load Duration Factors for ASD

| Load Type | Load Duration | Recommended $C_D$ Value | $\frac{C_D}{1.6}$ |
|---|---|---|---|
| Permanent (dead load) | Lifetime | 0.9 | 0.56 |
| Normal | Ten years | 1.0 | 0.63 |
| Occupancy (live load) [1] | Ten years to seven days | 1.0 to 1.25 | 0.63 to 0.78 |
| Snow [2] | One month to seven days | 1.15 to 1.25 | 0.72 to 0.78 |
| Temporary construction | Seven days | 1.25 | 0.78 |
| Wind and seismic [3] | Ten minutes to one minute | 1.6 to 1.8 | 1.0 to 1.13 |
| Impact | One second | 2.0 | 1.25 |

Source: Based on NDS•2.3.2 and NDS•Appendix B (AF&PA, 1997).
Notes:
[1] The NDS uses a live load duration of ten years ($C_D$ = 1.0). The factor of 1.25 is consistent with the time effect factor for live load used in the new wood LRFD provisions (AF&AP, 1996a).
[2] The NDS uses a snow load duration of one month ($C_D$ = 1.15). The factor of 1.25 is consistent with the time effect factor for snow load used in the new wood LRFD provisions (AF&PA, 1996a).
[3] The NDS uses a wind and seismic load duration of ten minutes ($C_D$ = 1.6). The factor may be as high as 1.8 for earthquake loads which generally have a duration of less than 1 minute with a much shorter duration for ground motions in the design level range.


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
0
1
2
3
4
5

The normal pdf, cdf are:

$$pdf(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2} \qquad cdf(x) = \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

and $E(X) = \mu \quad Var(X) = \sigma^2 \quad CV(X) = \frac{\sqrt{Var(X)}}{E(X)} = \frac{\sigma}{\mu}$

where $k = \sigma$ and $u = \mu$ in NBS SP577.

The lognormal pdf, cdf are:

$$pdf(x) = \frac{1}{x\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln x-\mu_1}{\sigma_1}\right]^2} \qquad cdf(x) = \frac{1}{2}\left[1 + erf\left(\frac{\ln x-\mu_1}{\sigma_1\sqrt{2}}\right)\right]$$

and $\sigma_1 = \sqrt{\ln\left(1 + (CV(X))^2\right)} \quad \mu_1 = \ln\left(\frac{E(X)}{\sqrt{1 + (CV(X))^2}}\right)$

where $k = \sigma_1$ and $u = \mu_1$ in NBS SP577.

The Weibull or Type III Extreme value pdf, cdf are:

$$pdf(x) = \frac{\alpha}{\eta}\left(\frac{x}{\eta}\right)^{\alpha-1} e^{-\left(\frac{x}{\eta}\right)^\alpha} \qquad cdf(x) = 1 - e^{-\left(\frac{x}{\eta}\right)^\alpha}$$

and $E(X) = \eta\Gamma\left(1 + \frac{1}{\alpha}\right) \quad Var(X) = \eta^2\left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \left(\Gamma\left(1 + \frac{1}{\alpha}\right)\right)^2\right] \quad CV(X) = \frac{\sqrt{\Gamma(1 + \frac{2}{\alpha}) - (\Gamma(1 + \frac{1}{\alpha}))^2}}{\Gamma(1 + \frac{1}{\alpha})}$

where $k = \alpha$ and $u = \eta$ in NBS SP577.

The gamma pdf, cdf are:

$$pdf(x) = \frac{\beta}{\Gamma(\alpha)}(\beta x)^{\alpha-1} e^{-\beta x} \qquad cdf(x) = \frac{(\beta x)^\alpha\, e^{-\beta x}}{\alpha\,\Gamma(\alpha)} \sum_{i=0}^{\infty} t_i$$

where $t_0 = 1$ then $t_i = t_{i-1}\,\frac{\beta x}{\alpha + i + 1}$

and $E(X) = \frac{\alpha}{\beta} \quad Var(X) = \frac{\alpha}{\beta^2} \quad CV(X) = \frac{1}{\sqrt{\alpha}}$

where $k = \alpha$ and $u = \beta$ in NBS SP577.

The Gumbel or Type I Extreme value pdf, cdf are:

$pdf(x) = \alpha e^{-\left(z + e^{-z}\right)} \qquad cdf(x) = e^{-e^{-z}}$ where $z = \alpha(x - u)$

and $E(X) = u + \frac{0.5772156649}{\alpha} \quad Var(X) = \frac{\pi^2}{6}\frac{1}{\alpha^2}$

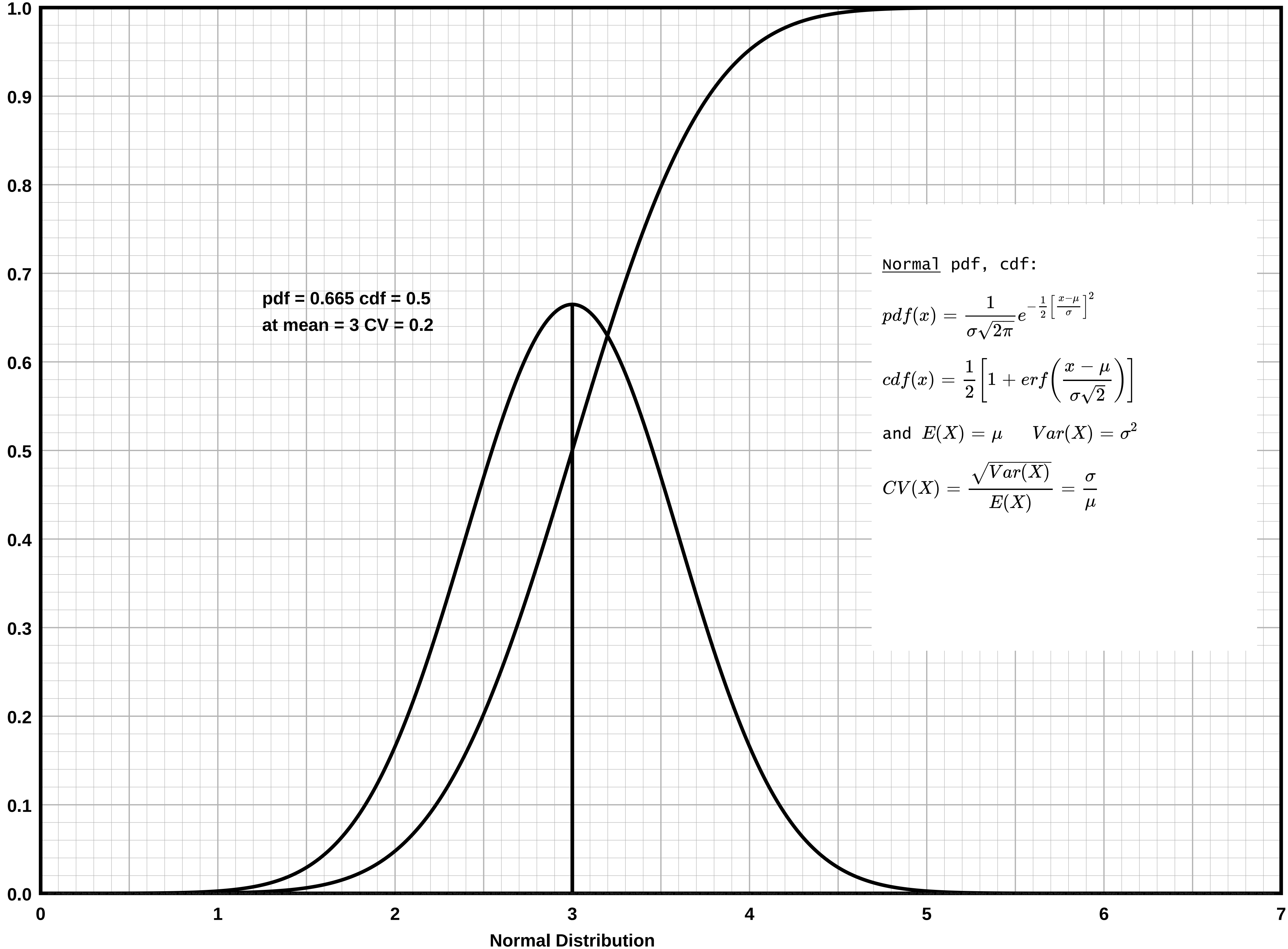
where $k = \alpha$ and $u = u$ in NBS SP577.

The Frechet or Type II Extreme value pdf, cdf are:

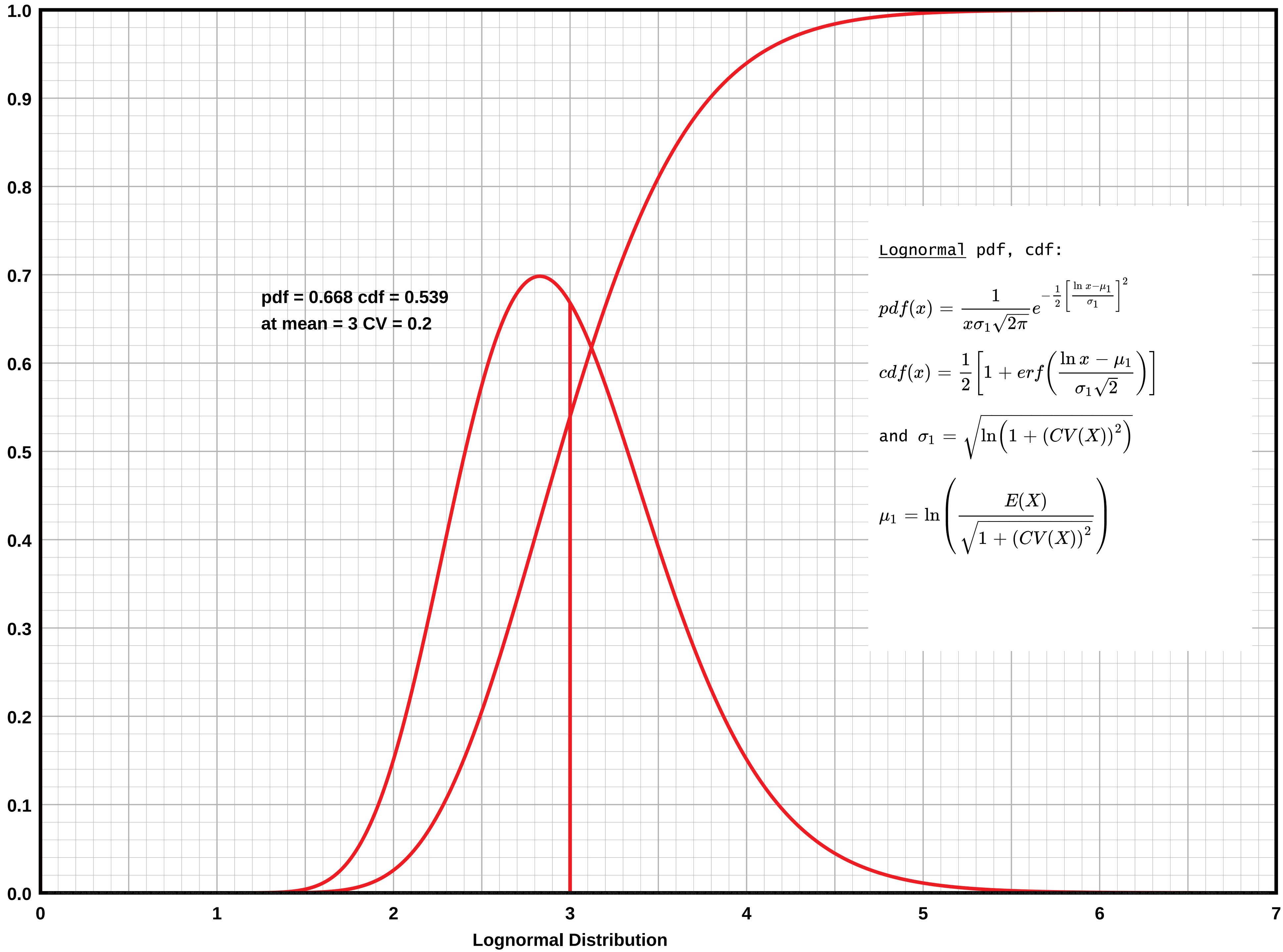
$$pdf(x) = \frac{\alpha}{s}\left(\frac{x}{s}\right)^{-\alpha-1} e^{-\left(\frac{x}{s}\right)^{-\alpha}} \qquad cdf(x) = e^{-\left(\frac{x}{s}\right)^{-\alpha}}$$

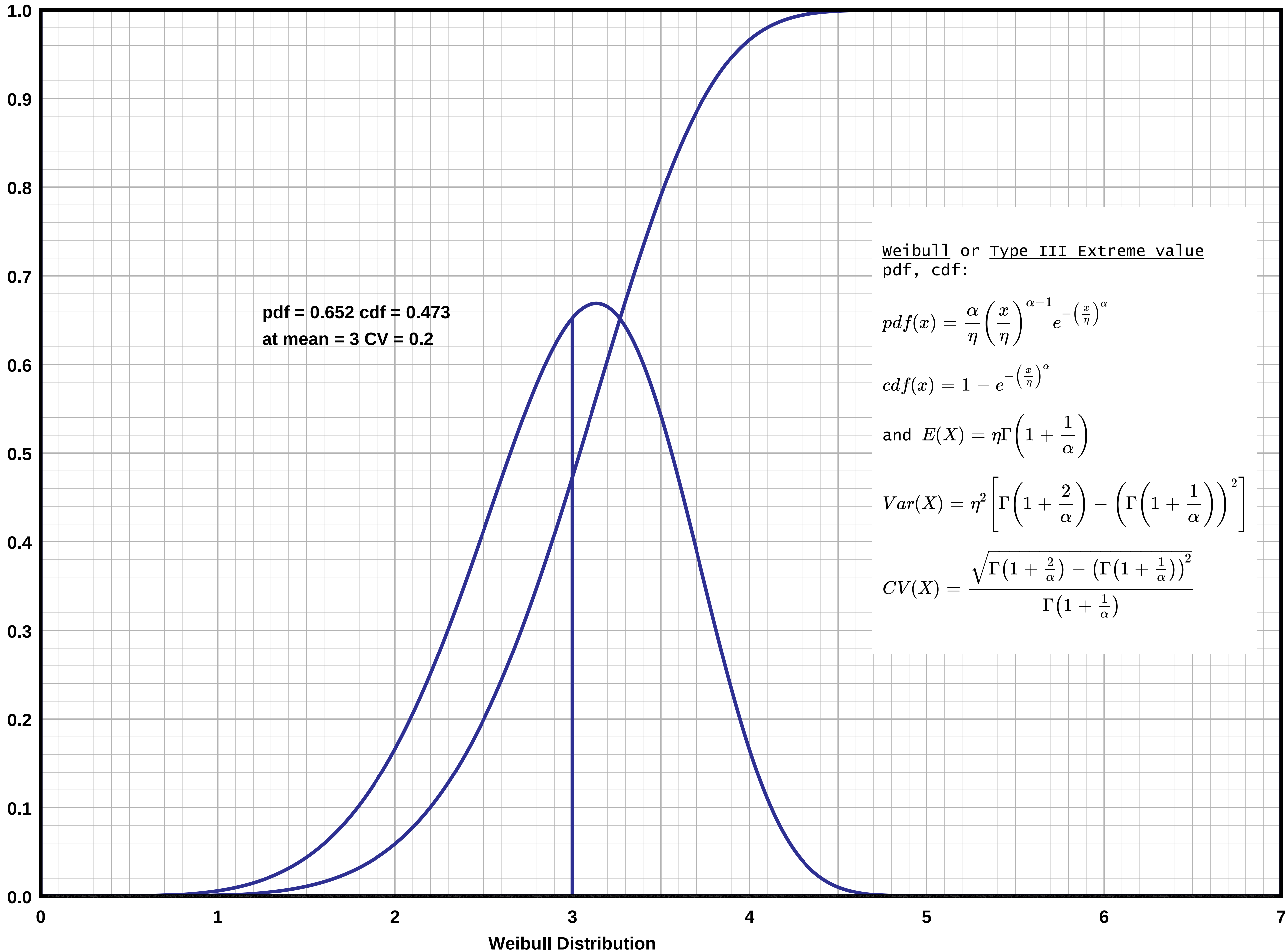
and $E(X) = s\Gamma\left(1 - \frac{1}{\alpha}\right) \quad Var(X) = s^2\left[\Gamma\left(1 - \frac{2}{\alpha}\right) - \left(\Gamma\left(1 - \frac{1}{\alpha}\right)\right)^2\right] \quad CV(X) = \frac{\sqrt{\Gamma(1 - \frac{2}{\alpha}) - (\Gamma(1 - \frac{1}{\alpha}))^2}}{\Gamma(1 - \frac{1}{\alpha})}$

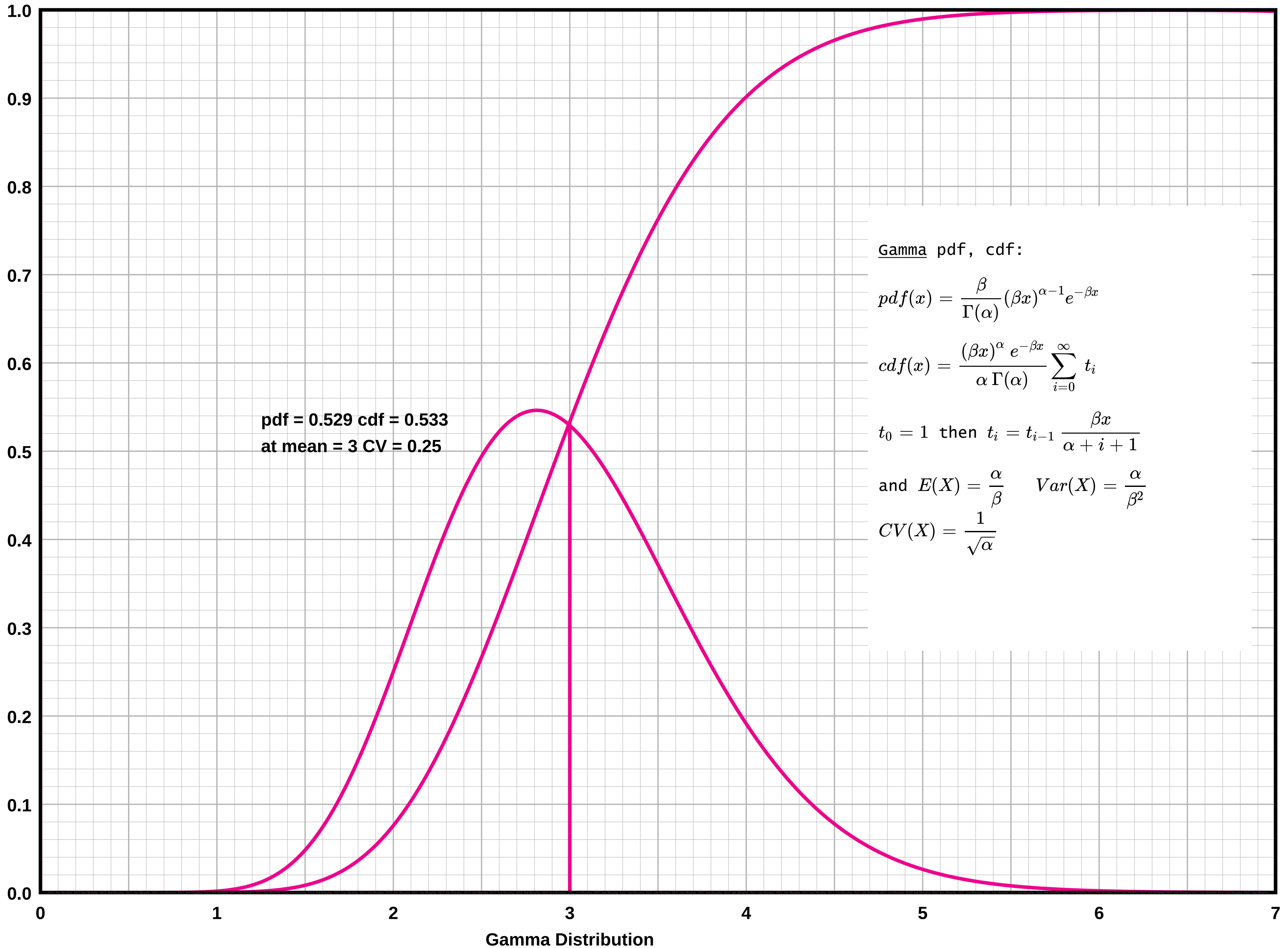
where $k = \alpha$ and $u = s$ in NBS SP577.



pdf = 0.665 cdf = 0.5
at mean = 3 CV = 0.2
Normal pdf, cdf:
$pdf(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$
$cdf(x) = \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$
and $E(X) = \mu$ $Var(X) = \sigma^2$
$CV(X) = \frac{\sqrt{Var(X)}}{E(X)} = \frac{\sigma}{\mu}$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
1
2
3
4
5
6
7
Normal Distribution


pdf = 0.668 cdf = 0.539
at mean = 3 CV = 0.2
Lognormal pdf, cdf:
$pdf(x) = \frac{1}{x\sigma_1\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln x - \mu_1}{\sigma_1}\right]^2}$
$cdf(x) = \frac{1}{2}\left[1 + erf\left(\frac{\ln x - \mu_1}{\sigma_1\sqrt{2}}\right)\right]$
and $\sigma_1 = \sqrt{\ln\left(1 + (CV(X))^2\right)}$
$\mu_1 = \ln\left(\frac{E(X)}{\sqrt{1 + (CV(X))^2}}\right)$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
Lognormal Distribution


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
Weibull Distribution
pdf = 0.652 cdf = 0.473
at mean = 3 CV = 0.2
Weibull or Type III Extreme value
pdf, cdf:
$pdf(x) = \frac{\alpha}{\eta}\left(\frac{x}{\eta}\right)^{\alpha-1} e^{-\left(\frac{x}{\eta}\right)^{\alpha}}$
$cdf(x) = 1 - e^{-\left(\frac{x}{\eta}\right)^{\alpha}}$
and $E(X) = \eta\Gamma\left(1 + \frac{1}{\alpha}\right)$
$Var(X) = \eta^2\left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \left(\Gamma\left(1 + \frac{1}{\alpha}\right)\right)^2\right]$
$CV(X) = \frac{\sqrt{\Gamma(1 + \frac{2}{\alpha}) - (\Gamma(1 + \frac{1}{\alpha}))^2}}{\Gamma(1 + \frac{1}{\alpha})}$




Gamma pdf, cdf:

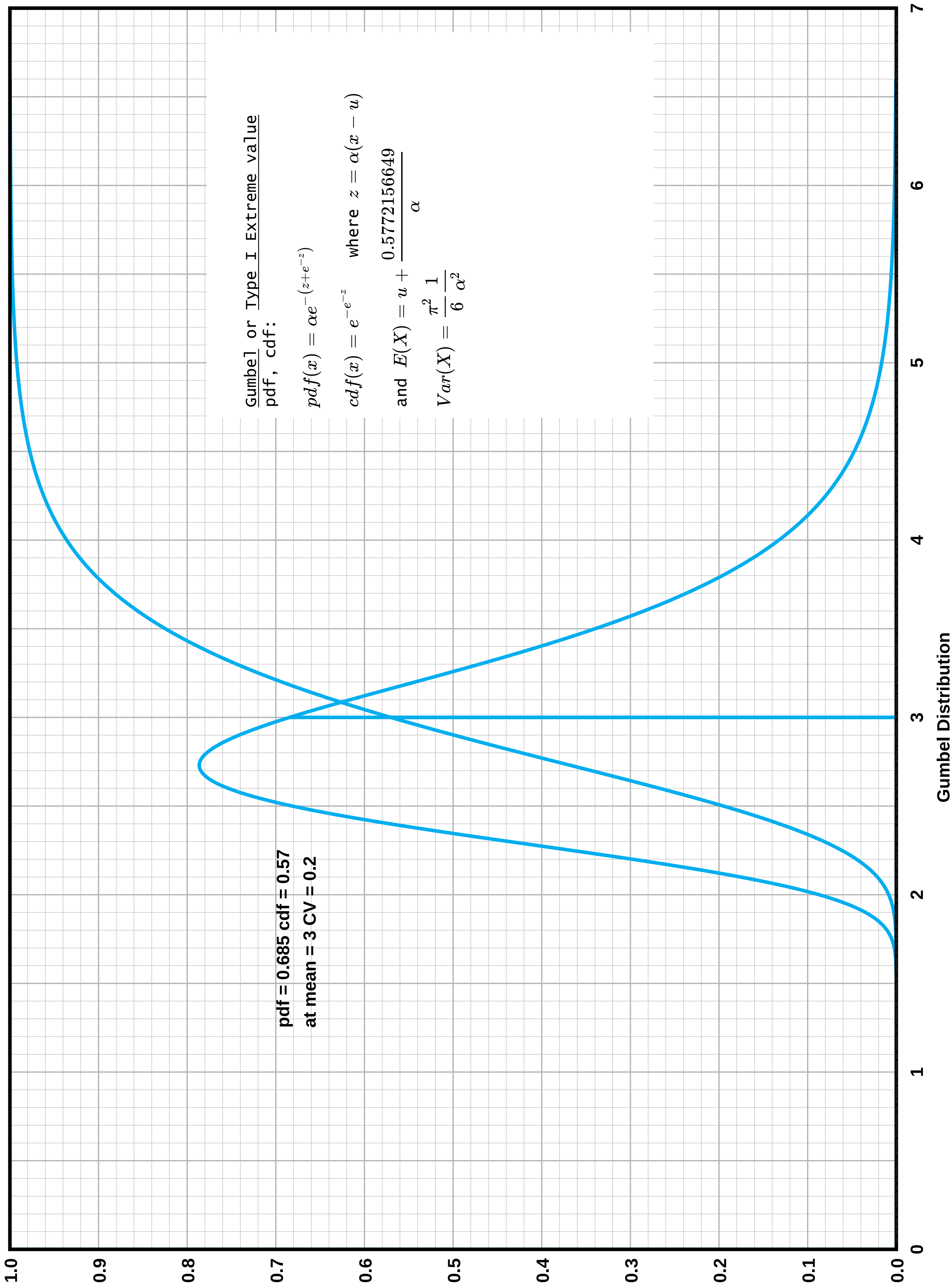
$$pdf(x) = \frac{\beta}{\Gamma(\alpha)}(\beta x)^{\alpha-1}e^{-\beta x}$$

$$cdf(x) = \frac{(\beta x)^{\alpha}\, e^{-\beta x}}{\alpha\,\Gamma(\alpha)}\sum_{i=0}^{\infty} t_i$$

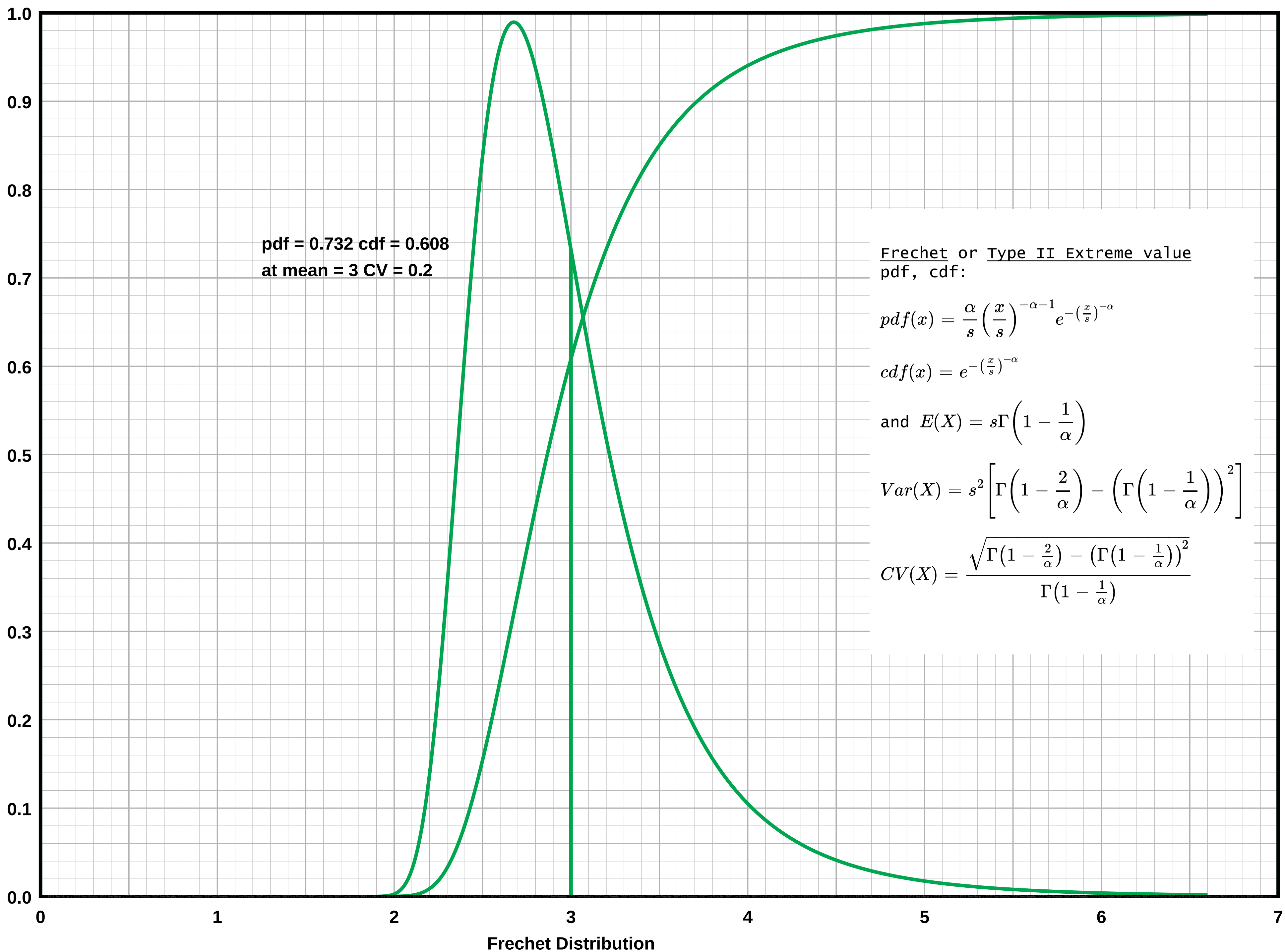
$t_0 = 1$ then $t_i = t_{i-1}\,\frac{\beta x}{\alpha + i + 1}$

and $E(X) = \frac{\alpha}{\beta}$ $\quad Var(X) = \frac{\alpha}{\beta^2}$

$$CV(X) = \frac{1}{\sqrt{\alpha}}$$



Gumbel or Type I Extreme value
pdf, cdf:
$pdf(x) = \alpha e^{-(z+e^{-z})}$
$cdf(x) = e^{-e^{-z}}$ where $z = \alpha(x-u)$
and $E(X) = u + \frac{0.5772156649}{\alpha}$
$Var(X) = \frac{\pi^2}{6}\frac{1}{\alpha^2}$
pdf = 0.685 cdf = 0.57
at mean = 3 CV = 0.2
Gumbel Distribution
0
1
2
3
4
5
6
7
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0



pdf = 0.732 cdf = 0.608
at mean = 3 CV = 0.2
Frechet or Type II Extreme value
pdf, cdf:
$pdf(x) = \frac{\alpha}{s}\left(\frac{x}{s}\right)^{-\alpha-1} e^{-\left(\frac{x}{s}\right)^{-\alpha}}$
$cdf(x) = e^{-\left(\frac{x}{s}\right)^{-\alpha}}$
and $E(X) = s\Gamma\left(1 - \frac{1}{\alpha}\right)$
$Var(X) = s^2\left[\Gamma\left(1 - \frac{2}{\alpha}\right) - \left(\Gamma\left(1 - \frac{1}{\alpha}\right)\right)^2\right]$
$CV(X) = \frac{\sqrt{\Gamma(1 - \frac{2}{\alpha}) - (\Gamma(1 - \frac{1}{\alpha}))^2}}{\Gamma(1 - \frac{1}{\alpha})}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
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3
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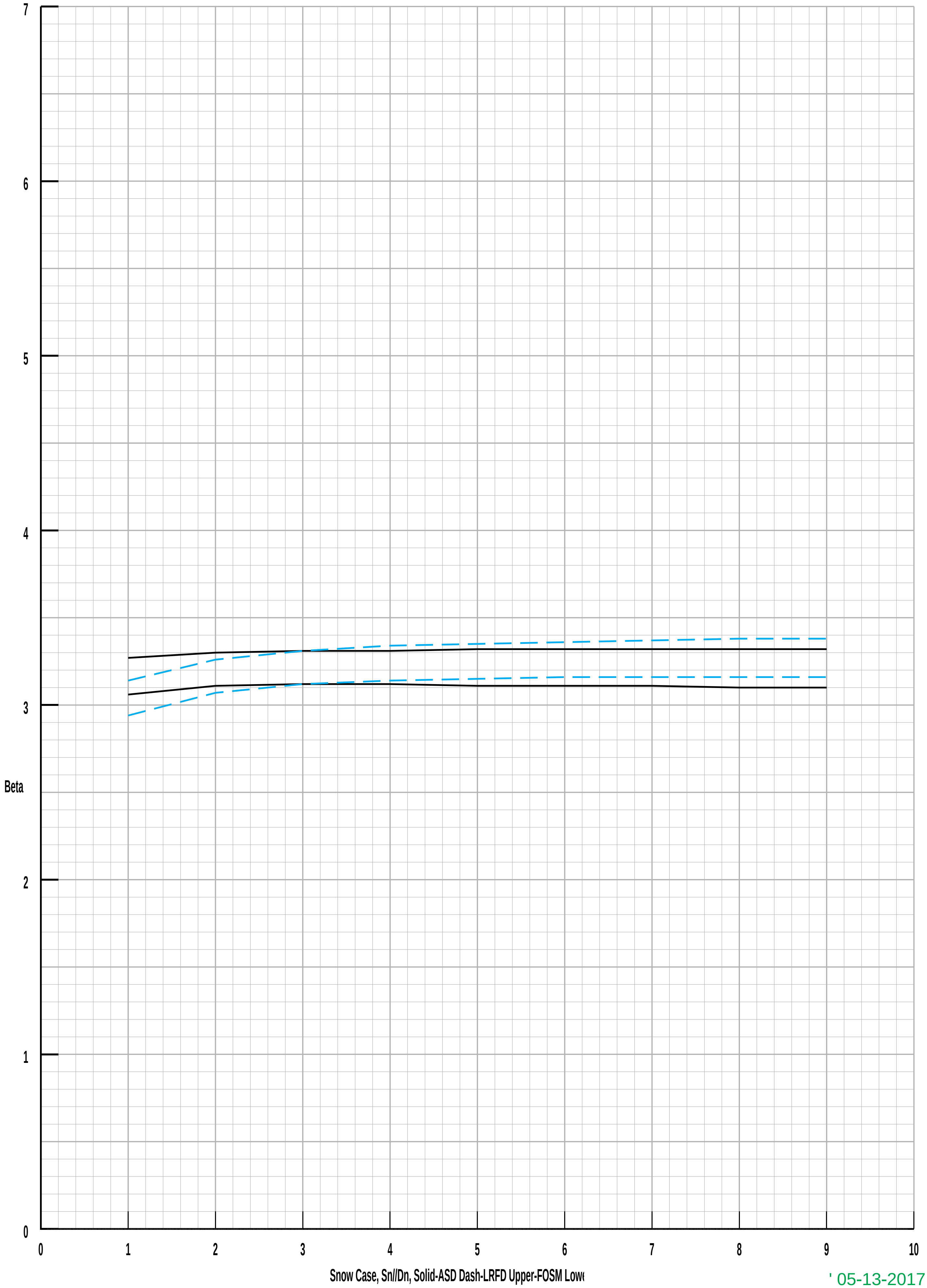
5
6
7
Frechet Distribution


7
6
5
4
3
2
1
0
Beta
0
1
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3
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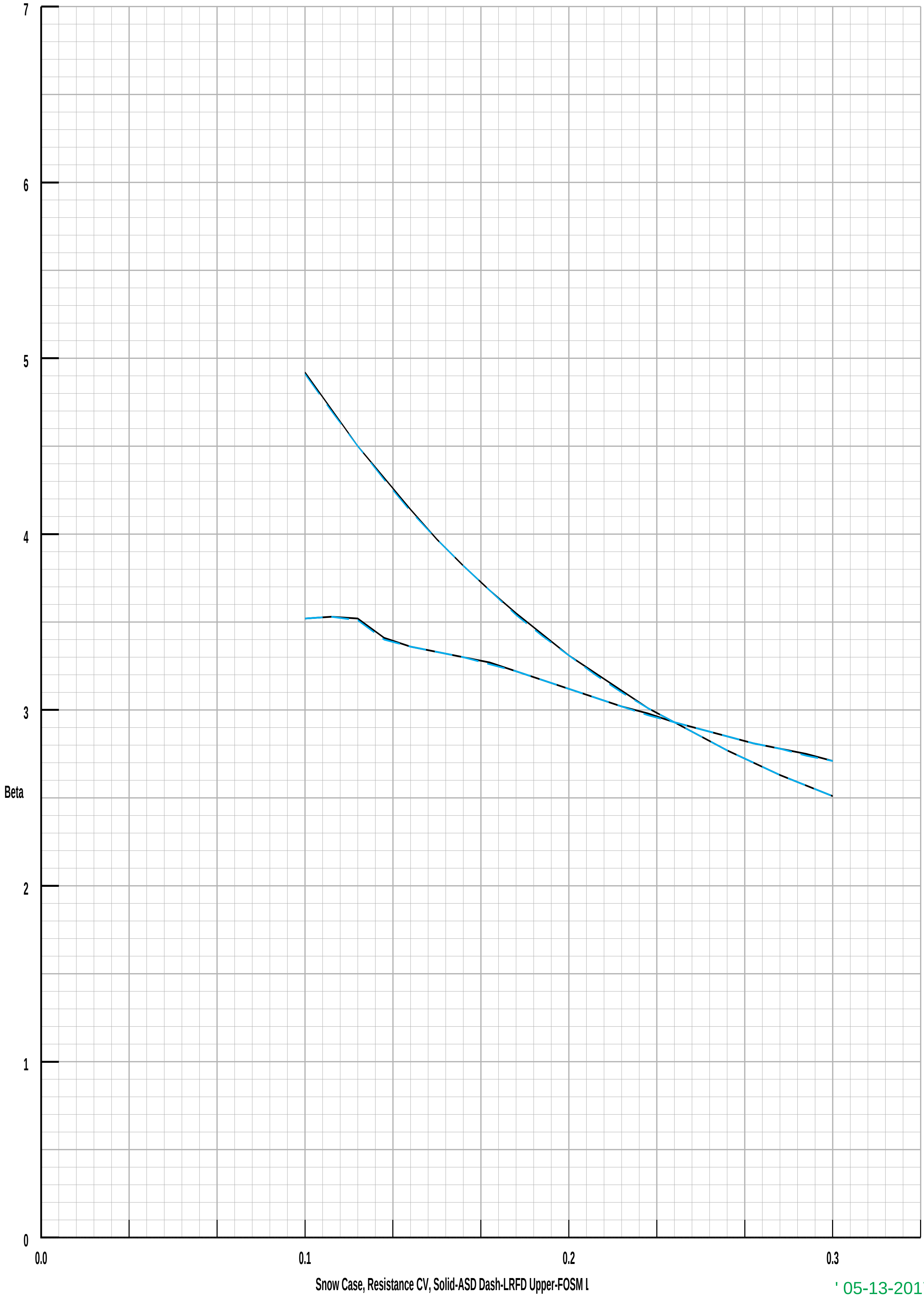
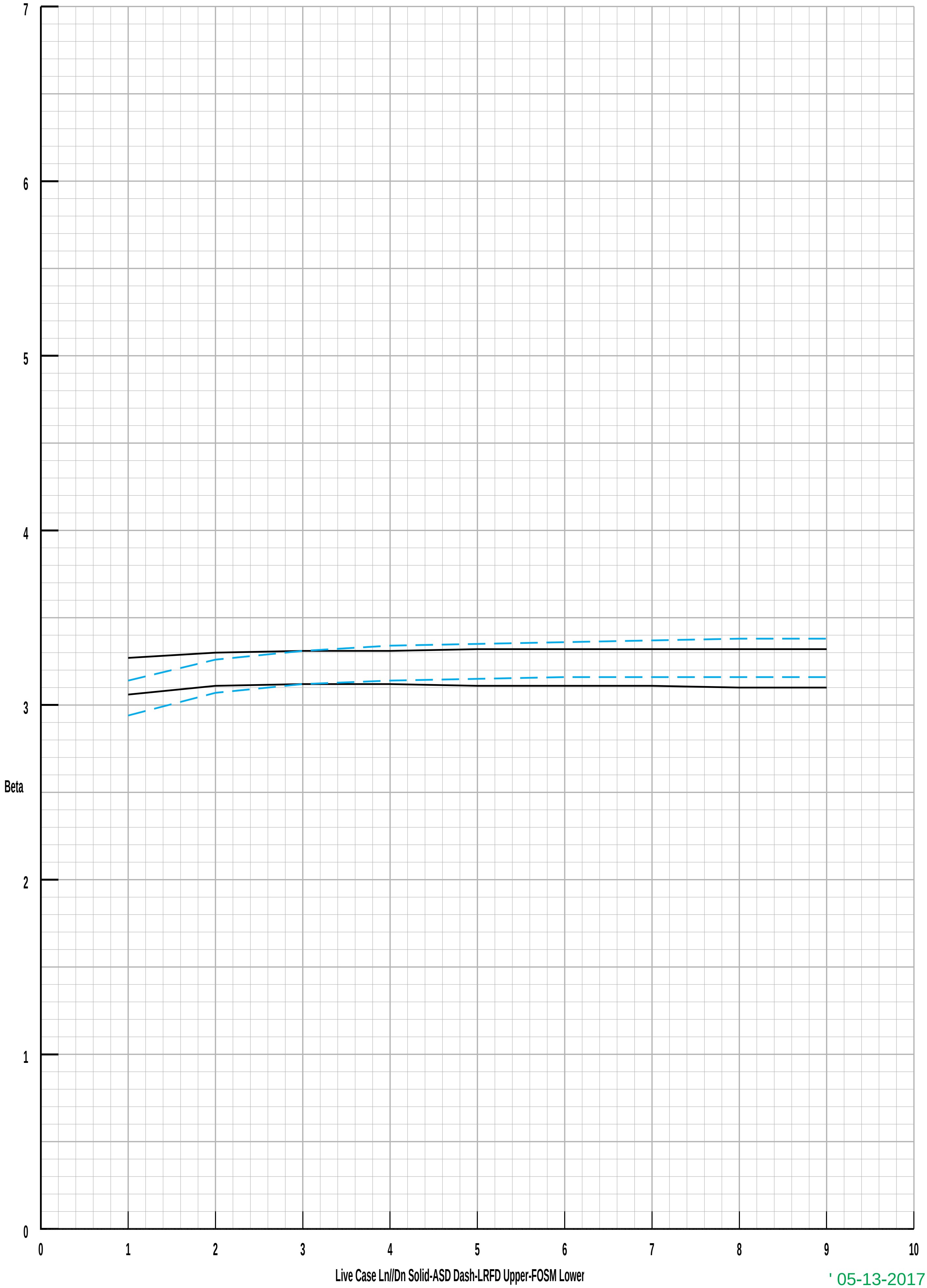
5
6
7
8
9
10
Snow Case, Sn//Dn, Solid-ASD Dash-LRFD Upper-FOSM Low


7
6
5
4
3
2
1
0
Beta
0.0
0.1
0.2
0.3
Snow Case, Resistance CV, Solid-ASD Dash-LRFD Upper-FOSM l



Beta
7
6
5
4
3
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1
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2
3
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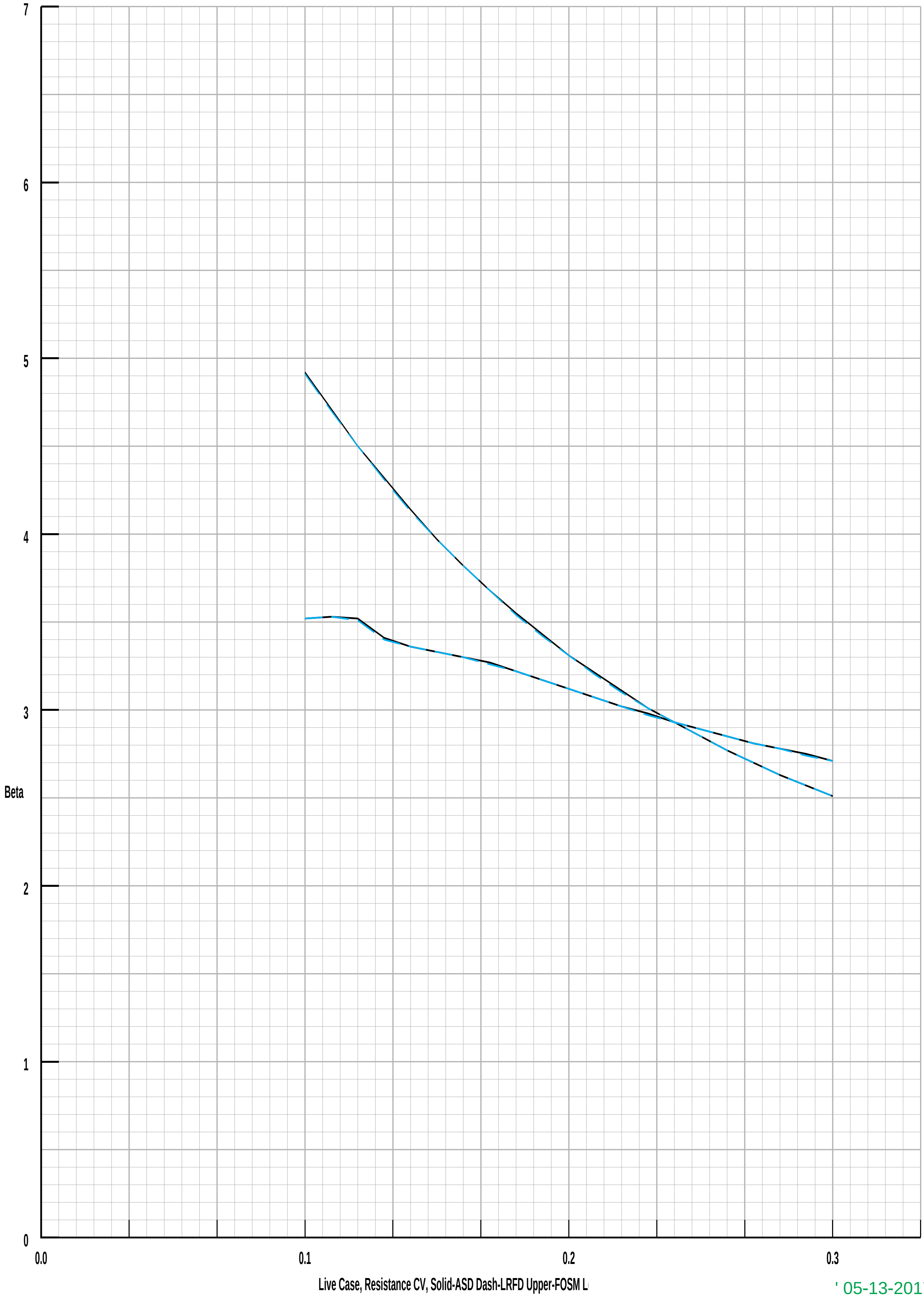
6
7
8
9
10
Live Case Ln//Dn Solid-ASD Dash-LRFD Upper-FOSM Lower


7
6
5
4
3
Beta
2
1
0
0.0
0.1
0.2
0.3
Live Case, Resistance CV, Solid-ASD Dash-LRFD Upper-FOSM L

```
% graph 1
/NAs2d [ 3.06 3.11 3.12 3.12 3.11 3.11 3.11 3.10 3.10] store
/NLs2d [ 2.94 3.07 3.12 3.14 3.15 3.16 3.16 3.16 3.16] store
/FAs2d [ 3.27 3.30 3.31 3.31 3.32 3.32 3.32 3.32 3.32] store
/FLs2d [ 3.14 3.26 3.31 3.34 3.35 3.36 3.37 3.38 3.38] store

% graph 3
/NAl2d [ 3.11 3.09 3.07 3.05 3.04 3.03 3.03 3.02 3.02] store
/NLl2d [ 2.98 3.03 3.05 3.06 3.06 3.07 3.07 3.07 3.07] store
/FAl2d [ 3.35 3.32 3.30 3.29 3.28 3.28 3.27 3.27 3.26] store
/FLl2d [ 3.21 3.27 3.29 3.30 3.31 3.31 3.31 3.32 3.32] store

% graph 2
/NAscv [ 3.52 3.53 3.52 3.41 3.36 3.33 3.30 3.27 3.22 3.17 3.12 3.07 3.02 2.98 2.93 2.89 2.85 2.81 2.78 2.75 2.71] store
/NLscv [ 3.52 3.53 3.51 3.40 3.36 3.33 3.30 3.26 3.22 3.17 3.12 3.07 3.02 2.97 2.93 2.89 2.85 2.81 2.78 2.74 2.71] store
/FAscv [ 4.92 4.71 4.50 4.32 4.14 3.97 3.82 3.68 3.55 3.43 3.31 3.21 3.11 3.01 2.93 2.85 2.77 2.70 2.63 2.57 2.51] store
/FLscv [ 4.91 4.70 4.50 4.31 4.13 3.97 3.82 3.68 3.54 3.42 3.31 3.20 3.10 3.01 2.93 2.85 2.77 2.70 2.63 2.57 2.51] store

% graph 4
/NAlcv [ 3.69 3.63 3.56 3.50 3.43 3.36 3.30 3.23 3.17 3.12 3.07 3.02 2.97 2.92 2.88 2.84 2.81 2.77 2.74 2.71 2.67] store
/NLlcv [ 3.67 3.61 3.54 3.48 3.41 3.34 3.28 3.22 3.16 3.10 3.05 3.00 2.96 2.91 2.87 2.83 2.79 2.76 2.73 2.69 2.66] store
/FAlcv [ 4.88 4.67 4.48 4.29 4.12 3.96 3.81 3.67 3.54 3.42 3.30 3.20 3.10 3.01 2.92 2.84 2.77 2.70 2.63 2.57 2.51] store
/FLlcv [ 4.84 4.64 4.45 4.27 4.09 3.93 3.79 3.65 3.52 3.40 3.29 3.19 3.09 3.00 2.91 2.83 2.76 2.69 2.62 2.56 2.50] store

%  first letter either NBS or FOSM
% second letter either ASD or LRFD
%  third letter either snow or live
% fourth,fifth letters either 2d (1 to 9 *n/Dn ratios, 9 values) or cv (0.10 to 0.30 by 0.01, 21 values)


% snowS2D.txt > sp57764.exe > snowS2D.htm > NBSbeta.exe > snowS2DNBS.dat  (NAs2d NLs2d)
% snowS2D.txt >                            FOSMbeta.exe > snowS2DFOSM.dat (FAs2d FLs2d)
% liveL2D.txt > sp57764.exe > liveL2D.htm > NBSbeta.exe > liveL2DNBS.dat  (NAl2d NLl2d)
% liveL2D.txt >                            FOSMbeta.exe > liveL2DFOSM.dat (FAl2d FLl2d)

% snowCV.txt > sp57764.exe > snowCV.htm > NBSbeta.exe > snowCVNBS.dat     (NAscv NLscv)
% snowCV.txt >                           FOSMbeta.exe > snowCVFOSM.dat    (FAscv FLscv)
% liveCV.txt > sp57764.exe > liveCV.htm > NBSbeta.exe > liveCVNBS.dat     (NAlcv NLlcv)
% liveCV.txt >                           FOSMbeta.exe > liveCVFOSM.dat    (FAlcv FLlcv)
```

APPENDIX F - COMPUTER PROGRAM

The purpose of this appendix is to describe the computer program used in the reliability analyses. Two separate problems can be handled with this program: (1) for a given design situation defined by a set of nominal load and resistance variables, calculate β (analysis), and (2) for a prescribed β and set of nominal loads, calculate the required nominal resistance and partial factors to be applied to the nominal value of each basic variable in the limit state equation (Level II design). The analysis procedure is summarized in Chapter 2 where an example calculation is also given.

The computer program can work with the following two-parametered probability distributions:

Normal

Lognormal

Gamma

Gumbel (Extreme Value Type I)

Frechet (Extreme Value Type II)

Weibull (Extreme Value Type III)

Additional distribution functions may be added if desired. In addition, several different forms of the limit state equation are allowed in the present version:

$$X_1 \mp X_2 \mp X_3 \ldots \mp X_n = 0 \qquad \text{(F.1)}$$

$$bX_1X_2 - X_3 = 0 \qquad \text{(F.2)}$$

$$X_1 - (X_2t + 6X_3)/bt^2 = 0; \; 0 \leq X_3/X_2t \leq 1/6 \qquad \text{(F.3a)}$$

$$X_1 - \frac{2}{3}\frac{P}{b}\left(\frac{t}{2} - \frac{X_3}{X_2}\right) = 0; \; 1/6 < X_3/X_2t \leq 1/2 \qquad \text{(F.3b)}$$

in which $X_i$ = basic variables and b,t = constants. Eq. F.1 is the common linear form of the limit state equation. Eq. F.2 is an alternate description of the limit state for a simple tension or bending member, in which $X_1$,$X_2$ = yield stress and section property, respectively, and $X_3$ = total load effect. Eqs. F.3a and F.3b describe the strength of an unreinforced masonry wall in compression plus bending, and were used to determine β at large vertical load eccentricities. Additional limit states could be added, if desired.

The linear form of the limit state equation was used for most of the calibrations and all of the Level II design calculations. The program assumes that $X_1$ is the resistance variable; $X_1$ may have a normal, lognormal, or Weibull probability distribution. When the design option is selected, the iteration is performed on $\bar{X}_1$. Variables $X_2$ - $X_6$ may have any of the six distributions listed previously. There is no restriction as to which X - variable describes which load.

The basic information required for either analysis or design options is the probability distribution for each variable, the ratio of mean to nominal value, $\bar{X}/X_n$, and the coefficient of variation $V_X$. For extreme value distributions I and II, the ratio of characteristic extreme to nominal, $u/X_n$, and the shape parameter k may be specified instead. There are additional input variables that describe the size of the problem, number of analysis or design situations to be considered, descriptors to assist in interpreting output, and so forth. Design situations may then be specified by a set of nominal loads and resistances. The means are then computed as $\bar{X} = (\bar{X}/X_n)\ X_n$ and the solution proceeds as described in Chapter 2. A detailed description of input data follows.

(1) NCASES

(2) HEADING FOR PROBLEM - ARBITRARY - MAXIMUM OF 72 CHARACTERS

(3) PROB N NG NLRFD BTA B T

(4) $(TYPE)_i$ $(DIST)_i$ $(\bar{X}/X_n)_i$ $(c.o.v.)_i$ $\gamma_i$

⋮

One card for each of "N" X-Variables.

⋮

(5) $X_{n_1}$ $X_{n_2}$ $X_{n_3}$ . . . . .

⋮

One card for each of "NLRFD" design situations

⋮

Card sets (2) - (5) may be repeated "NCASES" times.

The above parameters are defined as follows.

NCASES = number of problems - a problem is defined by a set of X-variables and their statistics (no limit).

PROB = ANALYS - calculate β for design situation

DESIGN - calculate partial factors for fixed β (Level II design).

N = number of X-variables in limit state equation

NG = designation of limit state;

= 1 - Eq. F.1, = 2 - Eq. F.2, = 3 - Eq. F.3

NLRFD = number of design situations in the problem (no limit)

BTA = reliability index $\beta$. If PROB = ANALYS, BTA is the initial guess at solution for $g(\beta) = 0$; if PROB = ANALYS, BTA is the target reliability for which Level II partial factors are sought.

B,T = constants. If NG = 1, they are not referenced. If NG = 2, B is an appropriate constant in Eq. F.2. If NG = 3, B,T = width, thickness of masonry wall.

$(TYPE)_i$ = user - defined description of $X_i$, e.g., "resist," "wind," etc.; maximum of 6 characters

$(DIST)_i$ = Probability distribution of $X_i$

= NORMAL - Normal distribution

= LOGNOR - Lognormal distribution

= GAMMMA - Gamma distribution

= GUMBEL - Type I Extreme value

= FRECHE - Type II Extreme value

= WEIBUL - Type III Extreme value

$(\bar{X}/X_n)_i$, $(c.o.v.)_i$ = mean-to-nominal, coefficient of variation

$\gamma_i$ = partial safety factor for $(X_n)_i$. If PROB = ANALYS, $\gamma_i$ is not needed as input.

$X_{n_1}, X_{n_2}, \ldots, X_{n_N}$ = nominal load and resistance variables which define each design situation. When the design option is selected, $X_{n_1} \cdot (\bar{X}_1/X_{n_1})$ is the initial guess at the solution for $g(\bar{X}_1) = 0$.

Table F.1 shows the input data used to calculate $\beta$'s for existing reinforced concrete beams under the D + L + W combination. Two values of $L_0/D$ were selected: 0.5 and 1.0. Four values of $W_n/D$ were considered at each $L_o/D$. Since $A_T = 400$ ft$^2$, $L_n = 0.68L_o$; $\bar{L}/L_n = 1.147$ for $D + L + W_{apt}$ and $\bar{L}/L_n = 0.353$ for $D + L_{apt} + W$. The statistics for maximum wind are $\bar{W}/W_n = 0.78$, $V_W = 0.37$, while for arbitrary-point-in-time wind, $u/W_n = -0.021$ and $k = 18.7$, the characteristic extreme and shape, respectively. The program is able to make the distinction by testing the magnitude of $(c.o.v.)_i$ in card (4); if the value in this location exceeds 1.0, the program assumes that $u/X_n$ and K were given.

A listing of the program follows. The addition of other limit states would require changes to subroutine GDGDX. Other distribution functions would require additional statements in subroutines CALC and PARAME. Separate subroutines must also be added to compute $F_{X_i}(X_i^*)$, $f_{X_i}(X_i^*)$, $\bar{X}_i^N$, $\sigma_{X_i}^N$ in accordance with Eqs. 2.24 (cf. subroutine FRECHE, which performs these operations for the Extreme Value Type II c.d.f.).

The program was written in Standard Fortran for a UNIVAC 1108 Exec 8 system. Several functions and subroutines from the UNIVAC scientific package were used which would have to be changed if the program were to run on another system:

TINORM ( ) - Inverse of standard normal distribution function: $X = \Phi^{-1}(p)$

GAMIN ( ) - Incomplete gamma function, necessary to evaluate gamma probability distribution function = $\int t^{n-1} e^{-t} dt/\Gamma(n)$

GAMMA ( ) - Complete gamma function.

Two cautionary notes are in order. First of all, the entire program (including UNIVAC - supplied routines) is written in single precision. When $\beta$ becomes large (say, 5 or greater) round off errors may occur when quantities such as $1 - p_f$ are computed. Second, convergence problems were encountered in the cases where $\beta\, \sigma^N_{X_i}/\bar{X}^N_i \approx 1.0$. This difficulty appears to be inherent to this particular reliability method, which replaces the actual non-normal variables with fitted normal variables prior to performing the reliability analysis. Consider, for example, the simple two-variable problem,

$$X_1 - X_2 = 0$$

The reliability analysis leads to a value of $\beta$:

$$\beta = \frac{\bar{X}^N_1 - \bar{X}^N_2}{[(\sigma^N_{X_1})^2 + (\sigma^N_{X_2})^2]^{1/2}}$$

where $\bar{X}^N_i$, $\sigma^N_{X_i}$ = mean, standard deviation in accordance with Eqs. 2.24. Conversely, the central factor of safety is,

$$\bar{X}^N_1/\bar{X}^N_2 = \frac{[V_1^2 + V_2^2\,(1 - \beta^2 V_1^2)]^{1/2}}{(1 - \beta^2 V_1^2)}$$

in which $V_i = \sigma^N_{X_i}/\bar{X}^N_i$. It is clear that as $\beta V_1 \rightarrow 1$, the central factor of safety increases without bound. There is no obvious way of circumventing this problem, and users of the method should be aware that it might occur. This was encountered in some of the analyses of masonry walls in "nearly pure" compression and of some connections where both variability in behavior and conservatism in practice are high.

Table F.1 - Sample Data Preparation

```
4
 R/C - FLEXURE - GRADE 40 - MED RHO - D + LI + WMAX   AI = 800 FT**2
   ANALYS          4          1          8          3.0
   RESIST     NORMAL      1.213      0.145       0.90
     DEAD     NORMAL      1.050       0.10       1.05
     LIVE     GAMMMA      0.353       0.55      1.275
     WIND     GUMBEL      0.780      0.370      1.275
    2.198        1.0       0.34       0.25
    2.357        1.0       0.34       0.50
    2.712        1.0       0.34       0.75
    3.066        1.0       0.34       1.00
    2.841        1.0       0.68       0.25
    2.841        1.0       0.68       0.50
    3.193        1.0       0.68       0.75
    3.543        1.0       0.68       1.00
 R/C - FLEXURE - GRADE 40 - MED RHO - D + LMAX + WI   AI = 800 FT**2
   ANALYS          4          1          8          3.0
   RESIST     NORMAL      1.213      0.145       0.90
     DEAD     NORMAL      1.050       0.10       1.05
     LIVE     GUMBEL      1.147       0.25      1.275
     WIND     GUMBEL     -0.021      18.70     1.275
    2.198        1.0       0.34       0.25
    2.357        1.0       0.34       0.50
    2.712        1.0       0.34       0.75
    3.066        1.0       0.34       1.00
    2.841        1.0       0.68       0.25
    2.841        1.0       0.68       0.50
    3.193        1.0       0.68       0.75
    3.543        1.0       0.68       1.00
 R/C - FLEXURE - GRADE 60 - MED RHO - D + LI + WMAX   AI = 800 FT**2
   ANALYS          4          1          8          3.0
   RESIST     NORMAL      1.103      0.110        0.90
     DEAD     NORMAL      1.050       0.10       1.05
     LIVE     GAMMMA      0.353       0.55      1.275
     WIND     GUMBEL      0.780      0.370      1.275
    2.198        1.0       0.34       0.25
    2.357        1.0       0.34       0.50
    2.712        1.0       0.34       0.75
    3.066        1.0       0.34       1.00
    2.841        1.0       0.68       0.25
    2.841        1.0       0.68       0.50
    3.193        1.0       0.68       0.75
    3.543        1.0       0.68       1.00
 R/C - FLEXURE - GRADE 60 - MED RHO - D + LMAX + WI   AI = 800 FT**2
   ANALYS          4          1          8          3.0
   RESIST     NORMAL      1.103      0.110       0.90
     DEAD     NORMAL      1.050       0.10       1.05
     LIVE     GUMBEL      1.147       0.25      1.275
     WIND     GUMBEL     -0.021      18.70      1.275
    2.198        1.0       0.34       0.25
    2.357        1.0       0.34       0.50
    2.712        1.0       0.34       0.75
    3.066        1.0       0.34       1.00
    2.841        1.0       0.68       0.25
    2.841        1.0       0.68       0.50
    3.193        1.0       0.68       0.75
    3.543        1.0       0.68       1.00
```

Table F.2 - Computer Program Listing

```
C   PROGRAM A58LF CALCULATES SAFETY INDEX BETA FOR GIVEN DESIGN
C   OR COMPUTES PARTIAL FACTORS FOR GIVEN BETA.
C   TYPE(I) = VARIABLE IDENTIFIER
C   DIST(I) = PROBABILITY DISTRIBUTION
C   U1(I), U2(I) = FIRST AND SECOND MOMENT PARAMETERS OF PROBABILITY
C   DISTRIBUTION, I.E., U1 = MEAN, OR CHARACTERISTIC EXTREME.
C   R(I), MX(I), CVX(I) = MEAN/NOMINAL RATIO, MEAN, COEFFICIENT
C   OF VARIATION.
C   XN(I) = NOMINAL DESIGN VALUES.
C   PF(I) = PARTIAL SAFETY FACTORS
      REAL MX, MXN, K
      DIMENSION TYPE(6), HEADER(12)
      COMMON/INSTAT/DIST(6), R(6), MX(6), CVX(6), K(6), U(6),
     1U1(6), U2(6)
      COMMON/CONSTS/N, NAL, NNR, NITAL, EPS, NG, B, T
      COMMON/NOMINL/XN(6), PF(6)
      COMMON/METRIC/X(6).MXN(6).SDXN(6).AL(6), BETA, BTA
      DATA/EPS,NAL,NNR,NITAL/0.001,100,20,20/
      READ 905, NCASES
905   FORMAT( )
      DO 1000 ICASE = 1, NCASES
C
C   READ IN BASIC PROGRAM VARIABLES
C
      READ 900, (HEADER(I), I = 1, 12)
      PRINT 902, (HEADER(I), I = 1, 12)
      READ 901, PROB, N, NG, NLRFD, BTA,  B, T
      IF(PROB .EQ. 'ANALYS') PRINT 802
      IF(PROB .EQ. 'DESIGN') PRINT 806, BTA
      DO 10 I = 1, N
10    READ 903, TYPE(I),DIST(I),U1(I),U2(I),PF(I)
      PRINT 800, (TYPE(I),I=1,N)
      PRINT 801, (DIST(I),I=1,N)
      IF(PROB .EQ. 'ANALYS') PRINT 803, (PF(I),I=1,N)
C
C   PERFORM ANALYSIS OR DESIGN CALCULATIONS FOR EACH OF FOLLOWING
C   NLRFD LOADING SITUATIONS.
C
      DO 1000 JJ = 1, NLRFD
      READ 904, (XN(I), I=1,N)
C   CIMPUTE DISTRIBUTION PARAMETERS OF PROGRAM VARIABLES FROM
C   NOMINAL DATA INPUT.
      CALL PARAME
C
C   BEGIN RELIABILITY CALCULATIONS.  ITERATIONS PERFORMED WITHIN
C   SUBROUTINE CALC.
      CALL CALC(PROB)
      PRINT 807, (XN(I),I=1,N)
      PRINT 804, (R(I),I=1,N)
      PRINT 805, (CVX(I),I=1,N)
      PRINT 808, (X(I),I=1,N)
      PRINT 809, (AL(I),I=1,N)
      IF(PROB .EQ. 'ANALYS') PRINT 810, BETA
      IF(PROB .EQ. 'DESIGN') PRINT 803, (PF(I),I=1,N)
1000  CONTINUE
C
```

```
900    FORMAT(12A6)
901    FORMAT(4X,A6,3I10,3F10.0)
902    FORMAT(////12A6//)
903    FORMAT(2(4XA6),3F10.0)
800    FORMAT(//' VARIABLE IN G( ) = 0',6(4X,A6))
801    FORMAT('           DISTRIBUTION',6(4X,A6))
802    FORMAT(//20X,'ANALYSIS'//)
803    FORMAT('       PARTIAL FACTORS',6F10.3)
804    FORMAT('            MEAN/NOMINAL',6F10.3)
805    FORMAT('                   C.O.V.',6F10.3)
806    FORMAT(//20X,'DESIGN - BETA =',F6.3//)
807    FORMAT(//'                    XN(I)',6F10.2)
808    FORMAT('                      X(I)',6F10.3)
809    FORMAT('                  ALPHA(I)',6F10.3)
810    FORMAT(15X,'***** BETA =',F6.2,' *****')
904    FORMAT(6F10.0)
       STOP
       END
       SUBROUTINE CALC(PROB)
C   CALC IS THE MAIN ROUTINE PERFORMING THE ITERATIONS OF STEPS 4 - 10
       EXTERNAL FBETA
       REAL MX, MXN, K
       DIMENSION XP(6), DGDX(6), A(6)
       COMMON/INSTAT/DIST(6), R(6), MX(6), CVX(6), K(6), U(6)
       COMMON/CONSTS/N, NAL, NNR, NITAL, EPS, NG
       COMMON/NOMINL/XN(6),PF(6)
       COMMON/METRIC/X(6),MXN(6),SDXN(6),AL(6), BETA, BTA
C   SET INITIAL CHECKING POINT VALUES EQUAL TO MEANS
       ITAL = 1
       BETA = BTA
       DO 10 I = 1, N
10     X(I) = MX(I)
99     IAL = 1
C   COMPUTE PARTIAL DERIVATIVES AT CHECKING POINT
100    CALL GDGDX(X,G,DGDX)
C
C   COMPUTE MEAN, STANDARD DEVIATION OF EQUIVALENT NORMAL DISTRIBUTION
C HAVING SAME CUMULATIVE AND DENSITY AT THE CHECKING POINT
       DO 17 I = 1, N
       IF(DIST(I) .EQ. 'NORMAL') GO TO 11
       IF(DIST(I) .EQ. 'LOGNOR') GO TO 12
       IF(DIST(I) .EQ. 'GAMMMA') GO TO 13
       IF(DIST(I) .EQ. 'GUMBEL') GO TO 14
       IF(DIST(I) .EQ. 'FRECHE') GO TO 15
       IF(DIST(I) .EQ. 'WEIBUL') GO TO 16
11     MXN(I) = MX(I)
       SDXN(I) = CVX(I)*MX(I)
       GO TO 17
12     CALL LOGNOR(X(I),U(I),K(I),MXN(I),SDXN(I))
       GO TO 17
13     CALL GAMMAL(X(I),U(I),K(I),MXN(I),SDXN(I))
       GO TO 17
14     CALL GUMBEL(X(I),U(I),K(I),MXN(I),SDXN(I))
       GO TO 17
15     CALL FRECHE(X(I),U(I),K(I),MXN(I),SDXN(I))
       GO TO 17
16     CALL WEIBUL(X(I),U(I),K(I),MXN(I),SDXN(I))
17     CONTINUE
C
C   COMPUTE DIRECTION COSINES FOR EACH VARIABLE
C
       SUM = 0.
       DO 20 I = 1, N
```

```
      A(I) = DGDX(I)*SDXN(I)
20    SUM = SUM + A(I)*A(I)
      SUM = SQRT(SUM)
      DO 21 I = 1, N
21    AL(I) = A(I)/SUM
C   COMPUTE NEW CHECKING POINT VALUES
      DO 22 I = 1, N
      XP(I) = X(I)
22    X(I) = MXN(I) - AL(I)*BETA*SDXN(I)
C   TEST WHETHER INTERIM ESTIMATES OF X(I) HAVE STABILIZED
      DO 24 I = 1, N
24    IF(ABS((X(I)-XP(I))/X(I)) .GT. 0.005) GO TO 23
      IF(PROB .EQ. 'ANALYS') GO TO 30
      IF(PROB .EQ. 'DESIGN') GO TO 31
23    IAL = IAL + 1
      IF(IAL .LE. NAL) GO TO 100
      GO TO 43
C
C   ANALYSIS PROBLEM.
C   COMPUTE VALUE OF BETA SUCH THAT G(    ) = 0.
C
30    BST = BETA
      CALL   NI(BETA,FBETA,BST,EPS,NNR,IER)
      IF(IER .EQ. 0) GO TO 25
      GO TO 41
C   TEST FOR CONVERGENCE OF SOLUTION
25    IF(ABS((BETA-BST)/BETA) .LT. 0.005) RETURN
      ITAL = ITAL + 1
      IF(ITAL .LE. NITAL) GO TO 99
      GO TO 42
C
C   DESIGN PROBLEM.
C   MODIFY MX(1) SO AS TO ACHIEVE G( ) = 0.
C
31    IF(ITAL .EQ. 1) GO TO 26
      DTHDG = (MX(1) - TH)/(G - GTH)
      TH = MX(1)
      GTH = G
      MX(1) = MX(1) - G*DTHDG
      IF(ABS((MX(1)-TH)/MX(1)) .LT. 0.005) GO TO 28
      GO TO 32
26    TH = MX(1)
      GTH = G
      IF(G) 27,28,29
27    MX(1) = 1.1*MX(1)
      GO TO 32
29    MX(1) = 0.9*MX(1)
32    ITAL = ITAL + 1
      IF(ITAL .GT. NITAL) GO TO 42
      CALL PARAMR
      GO TO 99
C
C   COMPUTE PARTIAL FACTORS FOR NOMINAL LOADS AND RESISTANCES.
C
28    XN(1) = MX(1)/R(1)
      DO 33 I = 1, N
33    PF(I) = X(I)/XN(I)
      RETURN
C
C   ERROR MESSAGES
C
41    PRINT 101
101   FORMAT(' SOLUTION OF G( ) = 0 NONCONVERGENT')
      CALL EXIT
```

```
42      PRINT 102
102     FORMAT(' SOLUTION OF N+1 EQUATIONS NONCONVERGENT')
        CALL EXIT
43      PRINT 103
103     FORMAT(' INTERIM SOLUTION FOR AL(I) NONCONVERGENT')
        CALL EXIT
        END
```

---

```
        SUBROUTINE FBETA(XX,F,DERF)
        REAL MXN
C   SUBROUTINE EVALUATES G( ) AND ITS DERIVATIVES WITH RESPECT TO BETA
        COMMON/CONSTS/N
        COMMON/METRIC/X(6),MXN(6),SDXN(6),AL(6),BETA,BTA
        DIMENSION X1(6), DGDX(6)
        DO 20 I = 1, N
20      X1(I) = MXN(I) - AL(I)*XX*SDXN(I)
        CALL GDGDX(X1,G,DGDX)
        F = G
        DERF = 0.0
        DO 21 I = 1, N
        DXDB = -AL(I)*SDXN(I)
21      DERF = DERF + DGDX(I)*DXDB
        RETURN
        END
```

---

```
        SUBROUTINE FRECHE(X,U,AL,MXN,SDXN)
        REAL MXN
        A = (U/X)**AL
        FC = EXP(-A)
        FD = FC*A*AL/X
        CALL XNORM(X,FC,FD,MXN,SDXN)
        RETURN
        END
```

---

```
        SUBROUTINE FXX(X,F,DERF)
        COMMON/FXNORM/FC1
        PHIX = 0.398942*EXP(-X*X/2.)
        F = PHIX*((1./X)-(1./X**3)+(3./X**5))-FC1
        DERF = -PHIX*(1.+15./X**6)
        RETURN
        END
```

---

```
        SUBROUTINE GAMMAL(X,LAM,K,MXN,SDXN)
        REAL LAM, K, MXN
        XX = LAM*X
        CALL GAMMA(K,GK,$21,$22)
        FC = GAMIN(XX,K)
        FD = LAM*XX**(K-1)*EXP(-XX)/GK
        CALL XNORM(X,FC,FD,MXN,SDXN)
        RETURN
21      WRITE(6,200)
200     FORMAT('***LOG10(GX) HAS BEEN COMPUTED***')
        GO TO 23
22      WRITE(6,201)
201     FORMAT('***ARGUMENT IS ZERO OR NEGATIVE***')
23      CALL EXIT
        END
```

---

```
        SUBROUTINE GDGDX(X,G,DGDX)
C   EVALUATE G( ) AND ITS DERIVATIVES AT POINT X(I).
        DIMENSION X(1),DGDX(1)
        COMMON/CONSTS/N,NAL,NNR,NITAL,EPS,NG,B,T
        COMMON/NOMINL/XN(6),PF(6)
        GO TO (1,2,3)     , NG
C   LIMIT STATE FUNCTION LINEAR IN BASIC VARIABLES.
1       G = X(1)
        DGDX(1) = 1.
```

```
      DO 22 I = 2, N
      IF(XN(I) .LT. 0.) GO TO 23
      DGDX(I) = -1.
      GO TO 22
23    DGDX(I) = 1.
22    G = G + DGDX(I)*X(I)
      RETURN
C
C  INSERT OTHER LIMIT STATES NEEDED
C
2     G = B*X(1)*X(2) - X(3)
      DGDX(1) = B*X(2)
      DGDX(2) = B*X(1)
      DGDX(3) = -1.
      RETURN
C
C  MASONRY WALL INTERACTION CURVE.
3     R = X(3)/(X(2)*T)
      IF(R .LT. 0.) GO TO 99
      IF(R .GT. 0.5) GO TO 99
      IF(R .GT. 0.166667) GO TO 31
C  FAILURE SURFACE 1 - UNCRACKED SECTION
      G = X(1) - (X(2)*T+6.*X(3))/(B*T*T)
      DGDX(1) = 1.
      DGDX(2) = -1./(B*T)
      DGDX(3) = -6./(B*T*T)
      RETURN
C  FAILURE SURFACE 2 - CRACKED SECTION.
31    A = .5*T - X(3)/X(2)
      C1 = 2./(3.*B*A)
      G = X(1) - C1*X(2)
      DGDX(1) = 1.
      DGDX(2) = -C1*(1. - X(3)/(A*X(2)))
      DGDX(3) = -C1/A
      RETURN
C
99    PRINT 101, R
101   FORMAT('  X(3)/(X(2)*T) =',F10.5,'  IS OUT OF RANGE')
      CALL EXIT
C
      END
```

---

```
      SUBROUTINE GUMBEL(X,U,AL,MXN,SDXN)
      REAL MXN
      A = EXP(-AL*(X-U))
      IF ( A.GT.7.5E-07 ) GO TO 1
      FC = A-A*A/2.
      FD = (AL*A)*(1.-FC)
      GO TO 2
    1 FC = EXP(-A)
      FD = AL*A*FC
    2 CALL XNORM(X,FC,FD,MXN,SDXN)
      RETURN
      END
```

---

```
      SUBROUTINE LOGNOR(X,U,AL,MXN,SDXN)
      REAL MXN
      SDXN = AL*X
      MXN = X*(1. - ALOG(X) + U)
      RETURN
      END
```

---

```
      SUBROUTINE PARAME
      REAL MX, K
      COMMON/INSTAT/DIST(6),R(6),MX(6),CVX(6),K(6),U(6),U1(6),U2(6)
      COMMON/CONSTS/N
      COMMON/NOMINL/XN(6),PF(6)
```

```
C   SUBROUTINE COMPUTES THE DISTRIBUTION PARAMETERS FOR THOSE
C   VARIABLES WHICH ARE NON-NORMAL
C   FROM THE MEANS, COEFFICIENTS OF VARIATION INPUT
C
      DO 9 I = 1, N
      R(I) = U1(I)
      CVX(I) = U2(I)
9     MX(I) = ABS(XN(I)*R(I))
C
C   LOAD VARIABLE PARAMETERS
      DO 20 I = 2, N
      IF(DIST(I) .EQ. 'NORMAL') GO TO 20
      IF(DIST(I) .EQ. 'LOGNOR') GO TO 12
      IF(DIST(I) .EQ. 'GAMMMA') GO TO 13
      IF(DIST(I) .EQ. 'GUMBEL') GO TO 14
      IF(DIST(I) .EQ. 'FRECHE') GO TO 15
      IF(DIST(I) .EQ. 'WEIBUL') GO TO 16
C
12    U(I) = ALOG(MX(I)/SQRT(1.+CVX(I)*CVX(I)))
      K(I) = SQRT(ALOG(1.+CVX(I)*CVX(I)))
      GO TO 20
C
13    K(I) = 1./(CVX(I)*CVX(I))
      U(I) = K(I)/MX(I)
      GO TO 20
C
14    IF(U2(I) .GT. 1.0) GO TO 140
      SDX = MX(I)*CVX(I)
      K(I) = 1.282/SDX
      U(I) = MX(I) - 0.5772/K(I)
      GO TO 20
140   U(I) = U1(I)*XN(I)
      K(I) = ABS(U2(I)/XN(I))
      MX(I) = U(I) + 0.5772/K(I)
      CVX(I) = 1.282/(K(I)*MX(I))
      R(I) = ABS(MX(I)/XN(I))
      GO TO 20
C
15    IF(U2(I) .GT. 1.) GO TO 150
      K(I) = 2.33/(CVX(I)**0.677)
      C1 = 1. - 1./K(I)
      CALL GAMMA(C1,GC1,$21,$22)
      U(I) = MX(I)/GC1
      GO TO 20
150   U(I) = U1(I)*XN(I)
      K(I) = U2(I)
      C1 = 1. - 1./K(I)
      C2 = 1. - 2./K(I)
      CALL GAMMA(C1,GC1,$21,$22)
      CALL GAMMA(C2,GC2,$21,$22)
      MX(I) = U(I)*GC1
      CVX(I) = SQRT(GC2/(GC1**2) - 1.)
      R(I) = ABS(MX(I)/XN(I))
      GO TO 20
C
16    K(I) = 1./(CVX(I)**1.08)
      C1 = 1. + 1./K(I)
      CALL GAMMA(C1,GC1,$21,$22)
      U(I) = MX(I)/GC1
20    CONTINUE
C
```

```
C   COMPUTE PARAMETERS FOR RESISTANCE VARIABLE
      ENTRY PARAMR
      IF(DIST(1) .EQ. 'NORMAL') GO TO 31
      IF(DIST(1) .EQ. 'LOGNOR') GO TO 32
      IF(DIST(1) .EQ. 'WEIBUL') GO TO 33
32    K(1) = SQRT(ALOG(1.+CVX(1)*CVX(1)))
      U(1) = ALOG(MX(1)/SQRT(1.+CVX(1)*CVX(1)))
      GO TO 31
33    K(1) = 1./(CVX(1)**1.08)
      C1 = 1. + 1./K(1)
      CALL GAMMA(C1,GC1,$21,$22)
      U(1) = MX(1)/GC1
31    RETURN
C
21    WRITE(6,200)
200   FORMAT('***LOG10(GX) HAS BEEN COMPUTED***')
      GO TO 23
22    WRITE(6,201)
201   FORMAT('***ARGUMENT IS ZERO OR NEGATIVE***')
23    CALL EXIT
      END
```

---

```
      SUBROUTINE   NI(X,FCT,XST,EPS,IEND,IER)
      IER = 0
      X = XST
      TOL = X
      CALL FCT(TOL,F,DERF)
      TOLF = 100.*EPS
      DO 6 I = 1, IEND
      IF(F)1,7,1
1     IF(DERF)2,8,2
2     DX = F/DERF
      XP = X
      X = X - DX
C
C   PREVENT NEGATIVE ROOT OR OVERSHOOTING
      IF(X .LE. 0.0) X = XP/10.
C
      TOL = X
      CALL FCT(TOL,F,DERF)
      TOL = EPS
      A = ABS(X)
      IF(A-1.)4,4,3
3     TOL = TOL*A
4     IF(ABS(DX) - TOL) 5,5,6
5     IF(ABS(F) - TOLF) 7,7,6
6     CONTINUE
      IER = 1
7     RETURN
8     IER = 2
      RETURN
      END
```

---

```
      SUBROUTINE WEIBUL(X,U,AL,MXN,SDXN)
      REAL MXN
      A = (X/U)**AL
      FC = EXP(-A)
      FD = AL*A*FC/X
      FC = 1. - FC
      CALL XNORM(X,FC,FD,MXN,SDXN)
      RETURN
      END
```

```
      SUBROUTINE XNORM(X,FC,FD,MXN,SDXN)
      EXTERNAL  FXX
      COMMON/CONSTS/N,NAL,NNR,NITAL,EPS
      COMMON/FXNORM/FC1
      FC1 = FC
      REAL  MXN
      IF ( FC.GT.7.5E-07 ) GO TO 1
      XST = 4.8
      CALL RTNI (XX,FXX,XST,EPS,NNR,IER)
      GO TO 2
    1 XX = TINORM(FC,$21)
    2 SDXN = 0.398942*EXP(-XX*XX/2.)/FD
      MXN = X - XX*SDXN
      RETURN
21    WRITE(6,100) FC
100   FORMAT(10X,'****EXIT CALLED FROM XNORM - FC =',E15.5)
      CALL EXIT
      END
```

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An option for running sp57764.exe not present in NBS 577.

Since the limit state is always 1, the limit state flag
is repurposed to calculate the resistance nominal.
Three conditions must be met.

1. Must be in analysis mode

2. Must have the limit state flag = 7

3. Must have resistance and load partial factors, pf, specified

The program calculates the resistance nominal
(overriding the resistance nominal specified):

$$R_n(1) = \left[ \sum_{i=2}^{m} pf(i) X_n(i) \right] / pf(1)$$

```
=========== example input txt file below ==========
2
Factors of Safety 1.15/2.1
Analys 3 7 9 2.4
Rlfd Weibull  1.546  0.20 0.548
Dead normal 1.05 0.1 1.0
Snow freche 0.82 0.26 1.0
1.0 1.0 1.0
1.0 1.0 2.0
1.0 1.0 3.0
1.0 1.0 4.0
1.0 1.0 5.0
1.0 1.0 6.0
1.0 1.0 7.0
1.0 1.0 8.0
1.0 1.0 9.0
Lambda 0.8
Analys 3 7 9 2.4
Rlfd Weibull  1.546  0.20 0.823
Dead normal 1.05 0.1 1.2
Snow freche 0.82 0.26 1.6
1.0 1.0 1.0
1.0 1.0 2.0
1.0 1.0 3.0
1.0 1.0 4.0
1.0 1.0 5.0
1.0 1.0 6.0
1.0 1.0 7.0
1.0 1.0 8.0
1.0 1.0 9.0
```

```
' sp57764.exe   BETA calculation consistent with NBS SP 577

' Joseph F. Murphy, Ph.D.  2017

' compiled with PowerBASIC Console Compiler 6.01

' Note: Only does linear limit state    NBS 577 Equ. F.1

' After NBS_577 completes the AFOSM reliability analysis
' the following, upto 15, equations hold true:

' g() = 0 = x*(1) - x*(2) - x*(3) - x*(4)

'       1 = alpha(1)^2 + alpha(2)^2 + alpha(3)^2 + alpha(4)^2

'    BETA =       [ mu*(1) - mu*(2) - mu*(3) - mu*(4) ] /
'          SQRT[ sig*(1)^2 + sig*(2)^2 + sig*(3)^2 + sig*(4)^2 ]

'  multiply alpha times BETA times sig*

'     x*(1) = mu*(1) + alpha(1) BETA sig*(1)
'     x*(2) = mu*(2) + alpha(2) BETA sig*(2)
'     x*(3) = mu*(3) + alpha(3) BETA sig*(3)
'     x*(4) = mu*(4) + alpha(4) BETA sig*(4)

'  CDF_1[ x*(1) ] = CDF_Normal[ x*(1),mu*(1),sig*(1) ]
'  PDF_1[ x*(1) ] = PDF_Normal[ x*(1),mu*(1),sig*(1) ]

'  CDF_2[ x*(2) ] = CDF_Normal[ x*(2),mu*(2),sig*(2) ]
'  PDF_2[ x*(2) ] = PDF_Normal[ x*(2),mu*(2),sig*(2) ]

'  CDF_3[ x*(3) ] = CDF_Normal[ x*(3),mu*(3),sig*(3) ]
'  PDF_3[ x*(3) ] = PDF_Normal[ x*(3),mu*(3),sig*(3) ]

'  CDF_4[ x*(4) ] = CDF_Normal[ x*(4),mu*(4),sig*(4) ]
'  PDF_4[ x*(4) ] = PDF_Normal[ x*(4),mu*(4),sig*(4) ]

' ==============================================================================

' These 2 (3) functions replace, TINORM(), GAMIN(), GAMMA() in NBS 577

  DECLARE FUNCTION TINORM!(P!)  ' inverse of standard normal CDF

  DECLARE FUNCTION GAMIN!(X!,R!)  ' incomplete gamma function
'  if argument X! = zero, then return complete GAMMA function of parameter R!

  DECLARE FUNCTION erf!(X!)     ' need for normal cdf and lognormal cdf

80 FUNCTION PBMAIN  'OPTION BASE 1
   DEFSNG a-z
   DIM dat$(10),vname$(4),Xdist$(4),dist%(4),u1(4),u2(4),pf(4),Xn(4)' input
90 DIM c(4),R(4),CVx(4),Mx(4),U(4),K(4)              ' from parame
   DIM mustr(4),sigstr(4),al(4),Xstr(4),XstrP(4)
   DIM cdf(4),pdf(4)

120 zero = 0!

160 REM      6          2             1
170 REM   "Weibul"l or "Lognor"mal or "Normal"  for Resistance Variable
180 REM      1       4         3         4          4        4
190 REM   "Dead" or "Live" or "Lapt" or "Wind" or "Wapt" or "Wann"
200 REM          or "Snow" or "Sann" or "Quak"e  for Loads
210 REM                5         2         5

' 1=Normal, 2=Lognormal, 3=Gamma, 4=Gumbel Type I, 5=Freche Type II, 6=Weibull Type III

218 REM******** Input data ***************************************************
220 REM

    filen$ = COMMAND$ : filen$ = RTRIM$(filen$) : filen$ = LTRIM$(filen$)

223 CLS
```

```
    IF LEN(filen$)=0 THEN
      INPUT "DATA INPUT 8.3 char Filename? (i.e., filename.txt): ",FILEN$
      filen$ = RTRIM$(filen$) : filen$ = LTRIM$(filen$)
      IF LEN(filen$)=0 THEN END
    END IF

    FILEO$ = LEFT$(filen$,INSTR(filen$,"."))+"HTM"

    dt$ = DATE$ : tm$ = TIME$

225 OPEN FILEN$ FOR INPUT AS #3
    OPEN FILEO$ FOR OUTPUT AS #4

    PRINT #4,"<html><body><pre>"
    PRINT #4,"<font face='lucida console'>"
    PRINT #4,Filen$;" ==> nbs57764 => ";fileo$;"   ";dt$;" ";tm$
    PRINT #4,"==========================================================="

   OPEN FILEN$ FOR INPUT AS #3

' read all of input file, write all to output file

    INPUT #3,Ncasemax
    PRINT #4,ncasemax

FOR ncase=1 TO ncasemax

    LINE INPUT #3,headr$
    PRINT #4,headr$

    GOSUB parseLine

    Prob$ = UCASE$(dat$(1)) : IF LEFT$(Prob$,6)="ANALYS" THEN prob%=1 ELSE prob%=2
    NV% = VAL(dat$(2))
    NG% = VAL(dat$(3))          ' linear limit state equation
     IF prob%=2 THEN NG%=1
    NLRFD% = VAL(dat$(4))
    BETAt = VAL(dat$(5)) : IF BETAt <= 0 THEN BETAt = 3
    PRINT #4,USING$("\    \",LEFT$(Prob$,6));
    PRINT #4,USING$("_ ##",nv%,1,nlrfd%);
    PRINT #4,USING$("_ ##.##",BETAt)

    c(1) = 1 : c(2) = -1 : c(3) = -1 : c(4) = -1    ' these are dgdx in NBS 577
    pflag% = -1
    FOR i=1 TO nv%
       pf(i) = 0!

       GOSUB parseLine

       vname$ = dat$(1) : vname$ = LCASE$(vname$)
       MID$(vname$,1,1) = UCASE$(LEFT$(vname$,1))
        dist$ = UCASE$(dat$(2))
        IF dist$="GAMMA" THEN dist$="GAMMMA"
        u1 = VAL(dat$(3)) : u1 = VAL(USING$("_ ##.###",u1))
        u2 = VAL(dat$(4)) : u2 = VAL(USING$("_ ##.###",u2))
        pf = VAL(dat$(5)) : pf = VAL(USING$("_ ##.###",pf))
       PRINT #4,USING$("\  \",vname$);
       PRINT #4,USING$("_ \    \",dist$);
       PRINT #4,USING$("_ ##.###",u1,u2,pf)
       IF pf = 0! THEN pflag% = 0
       IF pf < 0! THEN c(i) = 1
       pf(i) = pf
    NEXT 'i

    FOR jj=1 TO NLRFD%
       FOR i=1 TO NV%           ' read Xn(i)'s nominals
         INPUT #3,xn
         Xn(i) = ABS(xn) : Xn(i) = VAL(USING$("_ ##.###",Xn(i)))
       NEXT 'i

    ' special NG% switch to determine Xn(1) from checking equation
```

```
    ' only works if there are rlf's and doing analysis

       IF (ng%=7) THEN
         IF (pflag%) THEN
          sum = 0!: FOR i=2 TO nv% : sum = sum + pf(i)*Xn(i) : NEXT 'i
          Xn(1) = sum/pf(1) : Xn(1) = VAL(USING$("_ ##.###",Xn(1)))
         END IF
       END IF

       FOR i=1 TO NV% : PRINT #4,USING$("_ ##.###",Xn(i)); : NEXT 'i
       PRINT #4,""
    NEXT 'jj

NEXT ncase

  PRINT #4,"==================== End of Input ======================"
  PRINT #4,""
  CLOSE #3  ' close input file to reread it

' reread input file

  OPEN FILEN$ FOR INPUT AS #3

    INPUT #3,Ncasemax
FOR ncase=1 TO ncasemax

    LINE INPUT #3,headr$
    PRINT #4,headr$

    GOSUB parseLine

    Prob$ = UCASE$(dat$(1)) : IF LEFT$(Prob$,6)="ANALYS" THEN prob%=1 ELSE prob%=2
    NV% = VAL(dat$(2)) : nv = nv%
    NG% = VAL(dat$(3))           ' linear limit state equation
     IF prob%=2 THEN NG%=1
    NLRFD% = VAL(dat$(4))
    BETAt = VAL(dat$(5)) : IF BETAt <= 0 THEN BETAt = 3

    c(1) = 1 : c(2) = -1 : c(3) = -1 : c(4) = -1    ' these are dgdx in NBS 577
    pflag% = -1
    FOR i=1 TO nv%
       pf(i)=0!

       GOSUB parseLine

       vname$(i) = LCASE$(dat$(1)) : MID$(vname$(i),1,1) = UCASE$(LEFT$(vname$(i),1))
        dist$ = UCASE$(dat$(2))
        u1(i) = VAL(dat$(3)) : u1(i) = VAL(USING$("_ ##.###",u1(i)))
        u2(i) = VAL(dat$(4)) : u2(i) = VAL(USING$("_ ##.###",u2(i)))
           pf = VAL(dat$(5)) :    pf = VAL(USING$("_ ##.###",pf))

        Xdist$(i) = dist$
       IF INSTR(dist$,"NORMAL") THEN dist%(i)=1
       IF INSTR(dist$,"LOGNOR") THEN dist%(i)=2
       IF INSTR(dist$,"GAMMMA") THEN dist%(i)=3
       IF INSTR(dist$,"GAMMA" ) THEN dist%(i)=3 : Xdist$(i)="GAMMMA" ' see NBS 577
       IF INSTR(dist$,"GUMBEL") THEN dist%(i)=4
       IF INSTR(dist$,"FRECHE") THEN dist%(i)=5
       IF INSTR(dist$,"WEIBUL") THEN dist%(i)=6

       IF pf = 0! THEN pflag% = 0
       IF pf < 0! THEN c(i) = 1
       pf(i) = pf
    NEXT 'i

'========================================================

    PRINT #4,""
    IF prob%=1 THEN
    PRINT #4,SPACE$(21);"ANALYSIS - find BETA for a given Xn(1)"
    ELSE
        PRINT #4,SPACE$(21);"DESIGN - find Xn(1) for a given BETA =";
```

```
        PRINT #4,USING$("##.##",BETAt)
    END IF
    PRINT #4,""

    PRINT #4," VARIABLE IN G( ) = 0";
    FOR i = 1 TO NV% : PRINT #4,USING$("_ _ _ _ \  \",LEFT$(vname$(i),4)); : NEXT 'i
    PRINT #4,""
    PRINT #4,"         DISTRIBUTION";
    FOR i =1 TO nv% : PRINT #4,USING$("_ _ \    \",LEFT$(Xdist$(i),6)); : NEXT 'i
    PRINT #4,""

IF pflag% THEN
    PRINT #4,"      PARTIAL FACTORS";
    FOR i=1 TO nv% : PRINT #4,USING$("_ _ ##.###",pf(i)); : NEXT 'i
    PRINT #4,""
END IF

    PRINT #4,""

' ==============================================================================

    FOR jj=1 TO NLRFD%

       BETA = BETAt         ' reset initial BETA for each LRFD case

       FOR i=1 TO NV%           ' read Xn(i)'s nominals
         INPUT #3,xn
         Xn(i) = ABS(xn) : Xn(i) = VAL(USING$("_ ##.###",Xn(i)))
       NEXT 'i

    ' special NG% switch to determine Xn(1) from checking equation
    ' only works if there are rlf's and doing analysis

       IF (ng%=7) THEN
         IF (pflag%) THEN
          sum = 0!: FOR i=2 TO nv% : sum = sum + pf(i)*Xn(i) : NEXT 'i
          Xn(1) = sum/pf(1) : Xn(1) = VAL(USING$("_ ##.###",Xn(1)))
         END IF
       END IF

239 '-----------------------------------------------------------------------

    GOSUB PARAME            ' get distribution parameters U(i)'s K(i)'s

    GOSUB CALCS             ' does BETA analysis or design
    IF LEN(ermsg$) THEN PRINT #4,ermsg$    ' print error message if error occurs

IF pflag% THEN              ' only calc phi if there are partial factors
    sum = 0!: FOR i=2 TO nv% : sum = sum + pf(i)*Xn(i) : NEXT 'i
    phiC = sum/Xn(1) : phiC = VAL(USING$("_ ##.###",phiC))
ELSE
    phiC = 0!
END IF

IF NG% = 1 THEN
 IF pflag% THEN
    PRINT #4,"      PARTIAL FACTORS";
    PRINT #4,"  <u>"; : PRINT #4,USING$("##.###",phiC);  : PRINT #4,"</u>";
    FOR i=2 TO nv% : PRINT #4,USING$("_ _ ##.###",pf(i)); : NEXT 'i
    PRINT #4,""
 END IF
END IF ' ng%

    PRINT #4," Nominal values Xn(i)";
    FOR i=1 TO nv% : PRINT #4,USING$("_ _ ##.###",Xn(i)); : NEXT 'i
    PRINT #4,""

    PRINT #4,"         MEAN/NOMINAL";
    FOR i=1 TO nv% : PRINT #4,USING$("_ _ ##.###",R(i)); : NEXT 'i
    PRINT #4,""

    PRINT #4,"               C.O.V.";
```

```
    FOR i=1 TO nv% : PRINT #4,USING$("_ _ ##.###",CVx(i)); : NEXT 'i
    PRINT #4,""

    PRINT #4,"Checking points X*(i)";
    FOR i=1 TO nv% : PRINT #4,USING$("_ _ ##.###",Xstr(i)); : NEXT 'i
    PRINT #4,""

    PRINT #4,"             ALPHA(i)";
    FOR i=1 TO nv% : PRINT #4,USING$("_ _ ##.###",al(i)); : NEXT 'i
    PRINT #4,""

    IF prob%=1 THEN                     ' BETA is found in ANALYSIS
    PRINT #4,SPACE$(18);"***** BETA =<b>"; :PRINT #4,USING$("##.##",BETA);
        PRINT #4,"</b> *****"
    ELSE
        Xn(1) = Mx(1)/R(1)
        PRINT #4," NEW? PARTIAL FACTORS";
        FOR i=1 TO nv% : PRINT #4,USING$("_ _ ##.###",Xstr(i)/Xn(i)); : NEXT 'i
        PRINT #4,""
                            ' Xn(1) is found in DESIGN
    PRINT #4,SPACE$(18);"**** Xn(1) =<b>"; :PRINT #4,USING$("##.###",Xn(1));
        PRINT #4,"</b> ****"
    END IF

' anything in [ brackets ] is added output which NBS 577 does not print
    FOR i=1 TO nv% : GOSUB fromNBS577 : cdf(i)=FC : pdf(i)=FD : NEXT 'i

    PRINT #4,""
    PRINT #4,"[          cdf(X*(i)) ";
    FOR i=1 TO nv% : PRINT #4,USING$("###.####";cdf(i)); : NEXT 'i
    PRINT #4," ]"

    PRINT #4,"[          pdf(X*(i)) ";
    FOR i=1 TO nv% : PRINT #4,USING$("###.####";pdf(i)); : NEXT 'i
    PRINT #4," ]"

    PRINT #4,"[Shape parameter k(i) ";
    FOR i=1 TO nv% : PRINT #4,USING$("###.####";k(i)); : NEXT 'i
    PRINT #4," ]"

    PRINT #4,"[Scale parameter u(i) ";
    FOR i=1 TO nv% : PRINT #4,USING$("###.####";u(i)); : NEXT 'i
    PRINT #4," ]"

    PRINT #4,"[Normalized means mu* ";
    FOR i=1 TO nv% : PRINT #4,USING$("###.####",mustr(i)); : NEXT 'i
    PRINT #4," ]"

    PRINT #4,"[Normalized s.d. sig* ";
    FOR i=1 TO nv% : PRINT #4,USING$("###.####",sigstr(i)); : NEXT 'i
    PRINT #4," ]"

        PRINT #4,USING$("[X_*(1)/Xn(1) _=##.####",Xstr(1)/Xn(1));
        PRINT #4,USING$(",###.####",Xn(1));   ' Xn(1) is found in DESIGN
        PRINT #4,USING$(" _= Xn(1)_,  BETA _=##.####",BETA); : PRINT #4," ]"

IF NG% = 1 THEN
  IF pflag% THEN
    PRINT #4,SPACE$(22);
    PRINT #4,USING$("[##.####",phiC); : PRINT #4,USING$(" _=PHIc_,PHIc/#.##",pf(1));
    PRINT #4,USING$("_=##.####",phiC/pf(1)); : PRINT #4," ]"
  END IF
END IF
        PRINT #4,"=="
        PRINT #4,""

NEXT jj

PRINT #4,"==========================================================="

NEXT ncase
  PRINT #4,"</font></pre></body></html>"
```

```
CLOSE #3,#4

GOTO 9999  ' end program

'====================================================================
'============ subroutines ===========================================

CALCS:  ' set initial checking point
               flgA% = 0        ' used in DESIGN
               flgB% = 0

               EPS= .0001 '.00001      ' NBS uses .008

                   SUM1=0!
                   SUM2=0!
                   FOR I=1 TO NV%
                       SUM1=SUM1+c(i)*mustr(i)
                       SUM2=SUM2+sigstr(i)*sigstr(i)
                   NEXT
                   BETA=SUM1/SQR(SUM2)       ' FOSM Beta

prob2:  ' loop point for "DESIGN" option

'   DESIGN is nothing more that a sequence of ANALYSIS calculations
'                 homing until the BETA is equal to the target BETA

        REM****** Calculate  initial x*'s from initial est. of BETA *******
'           and mu's, sig's from distributions

               SUMK=0!
               FOR I=1 TO NV%
                   al(i)=-c(i)*sigstr(i)
                   SUMK=SUMK+al(i)*al(i)
               NEXT
               SQRK = SQR(SUMK)

    ' NBS starts out with Xstr(i) = mustr(i) only, no BETA, no al(i)'s

               FOR I= 1 TO NV%
                   al(i)=al(i)/SQRK
                   Xstr(i)= mustr(i)+al(i)*BETA*sigstr(i)
               NEXT

'        Xstr(i)  below mean for resistance, above mean for loads

'    setups for each ANALYSIS calculation

               ERMSG$=""
               ITER=0  : iterMax=15 '25
               ITERB=0 : iterBmax=5 '10

    ' 1460 is loop point for "ANALYSIS" option

1460           REM***** Normalize non-Normal distributions ******

'         compute mean ,sdev, of equivalent nomal distrib
'         with same pdf=FD, cdf=FC at the checking points x*'s
     REM
     REM****** Normalize non-Normal distributions ****************************
1470 REM

1860 REM***** calculate new mu*'s,sig*'s ************************************
1870 REM
1880 FOR I=1 TO NV% : GOSUB fromNBS577 : NEXT 'i
' ==========================================================================

        REM****** calculate g, dg/dx's and al's ********************
'                 update x*s
          REM
               SUMK=0!
               FOR I=1 TO NV%
```

```
                al(i)=-c(i)*sigstr(i)         ' use new sig*s
                SUMK=SUMK+al(i)*al(i)
            NEXT
            SQRK = SQR(SUMK)
            FOR I= 1 TO NV%
                XstrP(i) = Xstr(i)
                al(i)=al(i)/SQRK
                Xstr(i)=mustr(i)+al(i)*BETA*sigstr(i) ' use new mu*s sig*s
            NEXT

       REM******* set BETA so that g(x*'s) = 0 **************
'                g() = 0 = x*(1) - x*(2) - x*(3) - ...

                SUM1=0!
                SUM2=0!
                BETAP=BETA
                FOR I=1 TO NV%
                    SUM1=SUM1-c(i)*mustr(i)
                    SUM2=SUM2+c(i)*al(i)*sigstr(i)
                NEXT
                BETA=SUM1/SUM2       ' set Beta

            ITER=ITER+1
            IF (iterB <1) AND (iter < 5) GOTO 1460     ' do at least 5 times

       REM       compare checking points x*s

            FOR i=1 TO nv%
              IF ABS(1!- XstrP(i)/Xstr(i)) > EPS THEN
                IF iter < iterMax THEN
                  GOTO 1460
                ELSE
                  ermsg$ = "checking points, x*s, failed to converge after"
                  ermsg$ =  ermsg$ +STR$(iterMax)+" iterations"
                END IF
              END IF
            NEXT 'i

' either iter=itermax or x*s converge

                iterB = iterB + 1
                IF iterB < 2 THEN iter=0 : ermsg$="" : GOTO 1460 ' do at least twice

                IF iterB < iterBmax THEN
                  IF ABS(1!-BETAP/BETA) < EPS GOTO around2
                    iter = 0      ' restart interation count on x*'s convergence
                    ermsg$ = ""
                    GOTO 1460
                ELSE
                    ermsg$ = ermsg$ + "Solution for BETA is nonconvergent"
                    GOTO around2
                END IF
around2:
    IF (prob% = 1) GOTO 1810        ' found BETA

' DESIGN OPTION change Xn(1) to move resistance distribution and achieve target BETA

    IF ABS(1! - BETA/BETAt) < EPS THEN GOTO 1810

    IF BETA > BETAt THEN flgA% = -1 : xn1A = Xn(1)   ' calculated BETA above target
    IF BETA < BETAt THEN flgB% = -1 : xn1B = Xn(1)   ' calculated BETA below target

    IF (flgB% AND flgA%) THEN
      xn1 = (xn1A + xn1B)/2 ' squeeze time, last two iterations bracket the answer
    ELSEIF (flgB%) THEN
      xn1 = xn1B * 1.1      ' increase Xn(1) until squeeze time
    ELSE   'flgA%
      xn1 = xn1A * 0.9      ' decrease Xn(1) until squeeze time
    END IF

    Xn(1) = xn1         ' slides resistance distribution up or down
```

```
     i = 1          ' just change resistance distrib parameters
     GOSUB PARAMall     ' changes Mx(1), U(1), K(1), mu*(1), sig*(1)
               ' does not change u1(1), u2(1)
    GOTO prob2

        REM******* finish *************************************
1810          REM
     Xstr(1)=mustr(1)+al(1)*BETA*sigstr(1)

1811 RETURN                     ' from CALCS

'=============================================================================
PARAME:    ' from NBS 577   modified with weibull 1/alpha interations
                           ' good for all distributions, load AND resistance

    FOR i=1 TO nv%
      GOSUB PARAMall        ' does not change u1(i), u2(i)
    NEXT 'i
    RETURN          ' from PARAME

PARAMall:
      R(i) = u1(i)           ' these u1, u2 values are read once and stored
      CVx(i) = u2(i)
      Mx(i) = ABS(Xn(i)*R(i))
      nb% = dist%(i)

     ON nb% GOTO 480,530,850,740,1160,575
'                 N   L   Ga  Gu   F   W
435 '---------------------------------------------------------------------------
480                                                                 'Normal
        U(i)=Mx(i) : K(i)=CVx(i)*Mx(i)      ' every distrib has U(i), K(i)
490    GOTO 1370
495 '---------------------------------------------------------------------------
530                                                                 'Lognormal
540     U(i)=LOG( Mx(i)/SQR(1!+CVx(i)*CVx(i)) ) : K(i)=SQR( LOG(1!+CVx(i)*CVx(i)) )
550    GOTO 1370
555 '---------------------------------------------------------------------------
850                                                                 'Gammma
940     K(i)=1!/(CVx(i)*CVx(i)) : U(i)=K(i)/Mx(i)
950    GOTO 1370
955 '---------------------------------------------------------------------------
740                                                              'Gumbel Type I
        IF (u2(i) > 1!) THEN
          U(i)=u1(i)*Xn(i)
          K(i)=ABS(u2(i)/Xn(i))
          Mx(i)=U(i)+0.57721566/K(i)
          CVx(i)=1.2825498/(K(i)*Mx(i))
          R(i)=ABS(Mx(i)/Xn(i))
        ELSE
          sdx = Mx(i)*CVx(i)
          K(i)=1.2825498/sdx : U(i)=Mx(i)-0.57721566/K(i)
        END IF
820    GOTO 1370
825 '---------------------------------------------------------------------------
1160                                                           'Freche Type II
        IF (u2(i) > 1!) THEN
          U(i)=u1(i)*Xn(i)
          K(i)=u2(i)
          C1=1!-1!/K(i)
          C2=1!-2!/K(i)
          gc1 = GAMIN!(zero,c1)
          gc2 = GAMIN!(zero,c2)
          Mx(i)=U(i)*gc1
          CVx(i)=SQR(gc2/(gc1*gc1) -1!)
          R(i)=ABS(Mx(i)/Xn(i))
        ELSE
    CV= CVx(i)
    AI = CV^(0.667)/2.33 : aia = 0 : aib = 0
    DO                    ' loop to find ai=1/alpha vs CVx
      C1 = 1!-AI
      C2 = 1!-2*AI
      gc1 = GAMIN!(zero,c1)
```

```
      gc2 = GAMIN!(zero,c2)
      COV=SQR(gc2-gc1*gc1)/gc1
      DELTA = COV-CV
      IF ABS(delta)<0.000001 THEN EXIT DO
      IF delta < 0 THEN
        aib = ai
      ELSE
        aia = ai
      END IF

      IF (aib > 0) AND (aia > 0) THEN
         ai = (aib + aia)/2
      ELSEIF aib > 0 THEN
         ai = aib * 1.2
      ELSE ' aia > 0
         ai = aia * 0.8
      END IF
    LOOP
    al = 1!/ai              ' NBS sets 1/alpha = (cvx)^(0.667)/2.33

          K(i) = AL
          C1=1!-ai
          gc1 = GAMIN!(zero,c1)
          U(i)=Mx(i)/gc1
        END IF
1210   GOTO 1370
1215 '--------------------------------------------------------------------------
575                               'Weibull Type III
    CV= CVx(i)
    AI = CV^1.08 : aia = 0 : aib = 0
    DO                      ' loop to find ai=1/alpha vs CVx(i)
      C1 = 1!+AI
      C2 = 1!+2*AI
      gc1 = GAMIN!(zero,c1)
      gc2 = GAMIN!(zero,c2)
      COV=SQR(gc2-gc1*gc1)/gc1
      DELTA = COV-CV
      IF ABS(delta)<0.000001 THEN EXIT DO
      IF delta < 0 THEN
        aib = ai
      ELSE
        aia = ai
      END IF

      IF (aib > 0) AND (aia > 0) THEN
         ai = (aib + aia)/2
      ELSEIF aib > 0 THEN
         ai = aib * 1.2
      ELSE ' aia > 0
         ai = aia * 0.8
      END IF
    LOOP
    al = 1!/ai              ' NBS sets 1/alpha = (cvx)^(1.08)

        K(i) = AL
584     C1=1!+1!/K(i)
        gc1 = GAMIN!(zero,c1)
630     U(i)=Mx(i)/gc1
650    GOTO 1370
655 '---------------------------------------------------------------------------
1370
      mustr(i) = Mx(i)      ' starting mu*'s and sig*'s
      sigstr(i) = Mx(i)*CVx(i)  '    to compute starting checking points, x*'s
      RETURN            ' from PARAMall

'============================ parame, paramAll end ===============================
parseLine:
       LINE INPUT #3,work$ : work$=RTRIM$(work$) : work$=LTRIM$(work$) + " "
       FOR ii=1 TO 10 : dat$(ii)="" : NEXT 'ii

       FOR ii=1 TO 10
         n = INSTR(work$," ")
```

```
          IF n = 1 THEN EXIT FOR
          dat$(ii) = LEFT$(work$,n-1)
          work$ = RIGHT$(work$,LEN(work$)-n)
          work$ = RTRIM$(work$) : work$ = LTRIM$(work$) + " "
        NEXT 'ii
RETURN
' ===============================================================================
fromNBS577:

        nb%=dist%(i)
      '              Nor  Log  Gam  Gum  Fre  Wei
      ON nb% GOTO 3010,3020,3030,3040,3050,3060

3010 zz = (xstr(i) - u(i))/k(i)
     FC = 0.5 * (1! + erf!(zz/SQR(2)))
     FD = 0.39894228/k(i)*EXP(-0.5*zz*zz)
     GOTO 3080

3020 zz = (LOG(xstr(i)) - u(i))/k(i)
     FC = 0.5 * (1! + erf!(zz/SQR(2)))
     FD = 0.39894228/(xstr(i)*k(i))*EXP(-0.5*zz*zz)
     sigstr(i)=K(i)*Xstr(i) : mustr(i)=Xstr(i)*(1!-LOG(Xstr(i))+U(i))
     GOTO 3080

3030 xx = U(i)*Xstr(i)
     cgk = GAMIN!(zero,K(i))
     FC = GAMIN!(xx,K(i))       ' incomplete gamma
     FD = U(i)*xx^(K(i)-1)*EXP(-xx)/cgk
     GOTO 3070

3040 A = EXP(-K(i)*(Xstr(i)-U(i)))
     IF (A > 7.5E-07) THEN
       FC = EXP(-A)
       FD = K(i)*A*FC
     ELSE
       FC = A - A*A/2
       FD = (K(i)*A)*(1-FC)
     END IF
     GOTO 3070

3050 A = (U(i)/Xstr(i))^K(i)
     FC = EXP(-A)
     FD = FC*A*K(i)/Xstr(i)
     GOTO 3070

3060 A = (Xstr(i)/U(i))^K(i)
     FC = EXP(-A)
     FD = K(i)*A*FC/Xstr(i)
     FC = 1 - FC
     GOTO 3070

3070    'IF FC<1.5E-07 THEN FC=1.5E-07 ELSE IF 1!-FC<1.5E-07 THEN FC=1!-1.5E-07
        XX = TINORM!(FC)
        'IF FD<.0001 THEN FD=.0001

        sigstar= 0.39894228*EXP(-xx*xx/2)/FD : mustar=Xstr(i)-XX*sigstar

        IF mustar > 0! THEN         'need so mu* doesn't go negative
           mustr(i) = mustar
           sigstr(i) = sigstar
        END IF
3080 RETURN

' ===============================================================================

9999 END FUNCTION ' of reliability program

' ===============================================================================

FUNCTION TINORM!(P!) STATIC  ' by Peter John Acklam |err| < 1.15D-9

DEFDBL a-h,o-z
```

```
DEFINT i-n

DIM a(1 TO 6),b(1 TO 5),c(1 TO 6),d(1 TO 5)

IF NOT iflg THEN        ' store constants on 1st "CALL"
   iflg = -1
' The algorithm below assumes p is the input and x is the output.

'   Coefficients in rational approximations.

   a(1) = -3.969683028665376D+01
   a(2) =  2.209460984245205D+02
   a(3) = -2.759285104469687D+02
   a(4) =  1.383577518672690D+02
   a(5) = -3.066479806614716D+01
   a(6) =  2.506628277459239D+00

   b(1) = -5.447609879822406D+01
   b(2) =  1.615858368580409D+02
   b(3) = -1.556989798598866D+02
   b(4) =  6.680131188771972D+01
   b(5) = -1.328068155288572D+01

   c(1) = -7.784894002430293D-03
   c(2) = -3.223964580411365D-01
   c(3) = -2.400758277161838D+00
   c(4) = -2.549732539343734D+00
   c(5) =  4.374664141464968D+00
   c(6) =  2.938163982698783D+00

   d(1) =  7.784695709041462D-03
   d(2) =  3.224671290700398D-01
   d(3) =  2.445134137142996D+00
   d(4) =  3.754408661907416D+00

'   Define break-points.

   plow  = 0.02425
   phigh = 1 - plow

END IF

4000 p = CDBL(P!)

'   Rational approximation for lower region.

   IF (p < plow) THEN
      q = SQR(-2*LOG(p))
      x = (((((c(1)*q+c(2))*q+c(3))*q+c(4))*q+c(5))*q+c(6)) / _
            ((((d(1)*q+d(2))*q+d(3))*q+d(4))*q+1)

'   Rational approximation for upper region.

   ELSEIF (phigh < p) THEN
      q = SQR(-2*LOG(1-p))
      x = -(((((c(1)*q+c(2))*q+c(3))*q+c(4))*q+c(5))*q+c(6)) / _
             ((((d(1)*q+d(2))*q+d(3))*q+d(4))*q+1)

'   Rational approximation for central region.

   ELSE ' plow <= p <= phigh
      q = p - 0.5
      r = q*q
      x = (((((a(1)*r+a(2))*r+a(3))*r+a(4))*r+a(5))*r+a(6))*q / _
           (((((b(1)*r+b(2))*r+b(3))*r+b(4))*r+b(5))*r+1)
   END IF

   TINORM! = CSNG(x)

END FUNCTION 'TINORM!
' ============================================================================
```

```
  FUNCTION GAMIN!(X!,R!) STATIC

' From Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables
' U.S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series 55
' Ninth printing, November 1970, Library of Congress Catalog Card Number: 64-60036

  DEFDBL a-h,o-z
  DEFINT i-n

  DIM g(1 TO 26)

  IF NOT iflg THEN      ' store constants on 1st "CALL"
        iflg = -1

        g(1) =1.0D+0            ' see 6.1.34 Series Expansion
        g(2) = .5772156649015329D+0
        g(3) =-.6558780715202538D+0
        g(4) =-.420026350340952D-1
        g(5) = .1665386113822915D+0
        g(6) =-.421977345555443D-1
        g(7) =-.96219715278770D-2
        g(8) = .72189432466630D-2
        g(9) =-.11651675918591D-2
        g(10)=-.2152416741149D-3
        g(11)= .1280502823882D-3
        g(12)=-.201348547807D-4
        g(13)=-.12504934821D-5
        g(14)= .11330272320D-5
        g(15)=-.2056338417D-6
        g(16)= .61160950D-8
        g(17)= .50020075D-8
        g(18)=-.11812746D-8
        g(19)= .1043427D-9
        g(20)= .77823D-11
        g(21)=-.36968D-11
        g(22)= .51D-12
        g(23)=-.206D-13
        g(24)=-.54D-14
        g(25)= .14D-14
        g(26)= .1D-15
  END IF

5000    xx = CDBL(ABS(R!))      ' make sure gamma parameter is positive

5050 '==============================
     ' complete GAMMA
       IF XX = FIX(xX) THEN     ' R!=integer? then factorial!
              GA = 1#
              M1 = xx-1
              FOR k=2 TO m1
                ga = ga*k
              NEXT 'i
           ELSE
              IF xx > 1# THEN       ' see 6.1.15 Recurrence Formula
                z = xx
                m = FIX(z)
                r = 1#
                FOR k=1 TO m
                  r = r*(z-k)
                NEXT 'k
                z = z-m
              ELSE
                z = xx
              END IF

              GR = 0#
              FOR K=26 TO 1 STEP -1
                GR = (G(K)+GR)*Z
              NEXT 'k
              GA= 1#/GR

              IF xx > 1# THEN ga = ga * r
```

```
          END IF

    '===============================

    '   now ga = GAMMA(R!)

    IF X! = 0! THEN      ' we are done, with the complete gamma(r!)
       GAMIN! = CSNG(ga)

     '===============================
     ' incomplete GAMMA

        ELSE             ' if 0 < X!, we need the incomplete gamma
           A = CDBL(ABS(R!))
           X = CDBL(X!)

           term = 1#
           chf = 1#      ' see 6.5.12 Incomplete Gamma Function as a
           FOR k=1 TO 40        ' Confluent Hypergeometric Function
             term = term * x / (a+k)
             chf = chf + term           ' chf = M(1,1+a,x)
             IF ABS(term) < 1D-9 THEN EXIT FOR
           NEXT 'k

           gip = (x^A)/A*EXP(-x)*chf

           GAMIN! = CSNG(gip/ga) ' defined this way, gives gamin(x!,R!) = CDF

        END IF

END FUNCTION 'GAMIN!
' ==============================================================================

  FUNCTION erf!(X!) STATIC

' From Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables
' U.S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series 55
' Ninth printing, November 1970, Library of Congress Catalog Card Number: 64-60036

' This approximation is valid for x = 0. To use this approximation for negative x,
' use the fact that erf(x) is an odd function, so erf(x) = -erf(-x).

  DEFDBL a-h,o-z
  DEFINT i-n

  DIM a(1 TO 5)

  IF NOT iflg THEN       ' store constants on 1st "CALL"
        iflg = -1
        p = 0.3275911#       ' see approximation 7.1.26  (maximum error: 1.5×10-7)
        a(1) = 0.254829592#
        a(2) = -0.284496736#
        a(3) = 1.421413741#
        a(4) = -1.453152027#
        a(5) = 1.061405429#
  END IF

  x = CDBL(X!) : jflg = 1 : IF x < 0# THEN jflg = -1 : x = -x

  t = 1#/(1#+p*x)

  sum = 0 : FOR i=5 TO 1 STEP -1 : sum = (sum + a(i))*t : NEXT 'i

  erf! = jflg * (1-sum*EXP(-x*x))

END FUNCTION 'ERF!
' ==============================================================================
```